

Homework 6: MATH 4180

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Let γ be the circle of radius 5 centered at 0, oriented clockwise. Compute

$$\int_{\gamma} \frac{1}{\exp(\sin(\exp(z^2)))} dz.$$

2. Let γ be the circle of radius 1 centered at $5i$, oriented clockwise, and let $U = D(5i, 2)$. Define $\log : U \rightarrow \mathbb{C}$ by $\log z = \log r + i\theta$ where $z = re^{i\theta}$ with $\theta \in [0, 2\pi)$ and $r > 0$. Compute

$$\int_{\gamma} \log z dz.$$

3. Let γ be the ellipse given by the equation $x^2/9 + y^2/49 = 1$, oriented clockwise. Compute

$$\int_{\gamma} \frac{1}{z-2} dz.$$

4. Let γ be the circle of radius 12 centered at 0, oriented counter-clockwise. Prove that

$$\left| \int_{\gamma} \frac{\cos z}{z} dz \right| \leq 6\pi.$$

5. Prove or disprove: if $U \subset \mathbb{C}$ is open, $f : U \rightarrow \mathbb{C}$ holomorphic, and γ a C^1 curve in U , then

$$\overline{\int_{\gamma} f(z) dz} = \int_{\gamma} \overline{f(z)} dz.$$