Homework 07: MATH 4180

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Let γ be the unit circle, oriented counter-clockwise. Compute

$$\int_\gamma \frac{\sin e^z}{z} dz \quad \text{and} \int_\gamma \frac{\sin e^z}{z^2} dz.$$

2. Let $U \subset \mathbb{C}$ be an open set bounded by a simple closed C^1 curve γ. Let f be a holomorphic function on an open set V containing U and γ. Assume that f' is continuous on V. For any $z_0 \in U$, $z_0 \notin \gamma$, prove that

$$\int_\gamma \frac{f'(z)}{z - z_0} dz = \int_\gamma \frac{f(z)}{(z - z_0)^2} dz,$$

   where γ is oriented counter-clockwise.

3. Compute

$$\int_{S^1} \frac{\cos z}{z(z^2 + 8)} dz,$$

   where S^1 is oriented counter-clockwise.

4. Let γ be the circle of radius 10, oriented counter-clockwise. Compute

$$\int_\gamma \frac{z}{z^2 + 4} dz.$$

5. Compute

$$\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta) d\theta$$

   by consider the contour integral $\int_{S^1} \frac{e^z}{z} dz$.