Homework 9: MATH 4180

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Does the function $f : S^1 \to \mathbb{C}$ given by $f(z) = 1/z$ extend to a continuous function on the closed unit disc $\overline{B(0,1)}$ such that $f|_{B(0,1)}$ is holomorphic?
2. Does the function $f : S^1 \to \mathbb{R}$ given by $f(z) = \mathrm{Re}(1/z)$ extend to a continuous function on the closed unit disc $\overline{B(0,1)}$ such that $f|_{B(0,1)}$ is harmonic?
3. Prove that the series
$$\sum_{n=1}^{\infty} \frac{1}{n}$$
is divergent.

   Bonus (5 points): does $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ diverge or converge?
4. Let $u : \mathbb{C} \to \mathbb{R}$ be a harmonic function that is everywhere positive. Prove u is constant.
5. Let $U, V \subset \mathbb{C}$ be open, $f : U \to V$ holomorphic and $u : V \to \mathbb{R}$ harmonic. Prove that the composition $u \circ f : U \to \mathbb{R}$ is harmonic.