Homework 12: MATH 4180

Collaboration Policy: You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Compute
$$\int_{S^1(10)} \frac{z}{z^2+4}.$$
(This was also on Homework 07; this time do it using residue theory.)

2. Compute
$$\int_{S^1(0.1)} \frac{1+z^2}{1-\cos z} dz.$$

3. Let $f : D(0,1)^* \to \mathbb{C}$ be a holomorphic function such that the singularity at 0 is *not* removable. Prove that the function $g : D(0,1)^* \to \mathbb{C}$ given by $g(z) = \exp(f(z))$ has an essential singularity at 0.

4. Compute
$$\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx.$$
Hint: Use that $\cos x = \mathrm{Re}(e^{ix})$ for $x \in \mathbb{R}$.

5. Compute
$$\int_0^{2\pi} \frac{\cos \theta}{2+\cos \theta} d\theta.$$