

Homework 13: MATH 4180

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Compute

$$\int_{-\infty}^{\infty} \frac{x^{1/3}}{1+x^4} dx.$$

2. Let $f_n = \sum_{k=0}^n z^k/k!$. Fix $R > 0$. Prove that for n large enough (depending on R), f_n has no zeros in $D(0, R)$.
3. Let $U \subset \mathbb{C}$ open, and $p \in U$. Let $f : U - \{p\} \rightarrow \mathbb{C}$ be holomorphic, with a pole at p . Show that $f(U - \{p\})$ contains $\mathbb{C} - D(0, R)$ for some finite R .
4. (*Relevant material to be covered May 5.*) Find an example of a Mobius transformation f that takes $D(0, 1)$ onto $D(0, 1)$ and has no fixed point (i.e. there is no point $z \in D(0, 1)$ with $f(z) = z$).
5. (*Relevant material to be covered May 5.*) Let $D' := D(0, 1) - \{x + iy : x \geq 0\}$ be the half disc. Find a holomorphic bijection $f : D' \rightarrow D(0, 1)$.