Homework 2: MATH 4530

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly.

1. Find an open set $U \subset \mathbb{R}$, equipped with the standard topology, such that $U \subset (0, 1)$, and U is not a finite union of intervals.
2. Show that any open subset of $\mathbb{R}$ is a countable union of disjoint open intervals (we allow intervals of the form $(-\infty, a)$ and (b, ∞)).
3. Let X be a topological space and $A \subset X$. Suppose for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.
4. Show that if $\mathcal{A}$ is a basis for a topology on X, then the topology generated by $\mathcal{A}$ equals the intersection of all topologies that contain $\mathcal{A}$. Prove the same if $\mathcal{A}$ is a subbasis.
5. Is the open square $\{(x, y) : 0 < x, y < 1\} \subset \mathbb{R}^2$ a *countable* union of open balls?