

Homework 3 : MATH 4530

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a .pdf file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly.

1. Show that if Y is a subspace of X , and A is a subset of Y , then the topology A inherits as a subspace of Y is the same as the topology it inherits as a subspace of X .
2. Let $E = \{x : 0 \leq x < 1 \text{ and } 1/x \notin \mathbb{Z}_+\}$. Is E an open subset of $\mathbb{R}$? Is it open as a subset of $[0, 1]$, endowed with the subspace topology inherited from $\mathbb{R}$?
3. Show that the countable collection

$$\{(a, b) \times (c, d) : a < b \text{ and } c < d, \text{ and } a, b, c, d \in \mathbb{Q}\}$$

is a basis for $\mathbb{R}^2$.

4. Let X be a set equipped with the indiscrete topology. Give a simple characterization for convergence of a sequence x_n to x .
5. Show that if A is closed in X and B is closed in Y , then $A \times B$ is closed in $X \times Y$.
6. Show that any topology on X given by an order $\prec$ is Hausdorff.
7. Show that if X is Hausdorff iff the diagonal

$$\Delta = \{(x, x) : x \in X\}$$

is closed in $X \times X$.