Homework 5: MATH 4530

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly.

Notation: For $p \geq 1$ a real number, recall the ℓ_p metric on $\mathbb{R}^n$ given by

$$d_p(x, y) := \left(|x_1 - y_1|^p + \cdots + |x_n - y_n|^p\right)^{1/p}.$$

1. Let X be a set and Y a topological space. Let Y^X denote the set of functions f from X to Y. Describe a natural topology on Y^X such that a sequence of functions $f_1, f_2, \ldots$ converges to f iff $f_1(x), f_2(x), \ldots$ converges to $f(x)$ for all $x \in X$ (i.e. the f_i converge *pointwise* to f).

2. Verify that d_2 is a metric on $\mathbb{R}^n$.

3. For $p \geq 1$ a real number, show that the metric topology on $\mathbb{R}^n$ from d_p is the standard topology. (You can assume that d_p is in fact a metric.)

4. Let X be metrizable, and give $A \subset X$ the subspace topology. Show that A is metrizable.

5. Prove or disprove: If X, Y are non-empty topological spaces, and $X \times Y$ is metrizable, then X and Y are metrizable.

6. Find a sequence of continuous functions $f_n : \mathbb{R} \to \mathbb{R}$ that converges pointwise to some $f : \mathbb{R} \to \mathbb{R}$, with f *not* continuous.