Homework 7: MATH 4530

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly.

1. Let A be a compact subspace of the Hausdorff space X.

   (a) Let $x \in X - A$. Show that there exist disjoint open subsets U, V of X with $A \subset U$ and $x \in V$.

   (b) Let B be another compact subspace of X with $A \cap B = \emptyset$. Show that there exist disjoint open subsets U, V of X with $A \subset U$ and $B \subset V$.

2. Find a metric space (X, d) and a subspace $Y \subset X$ that is closed and bounded, but not compact.

3. Let X be the union of $\mathbb{R}^n$ and a new point called ∞. Define a topology on X whose basis consists of the open subsets of $\mathbb{R}^n$ (thought of as subsets of X), together with the sets of the form

$$\{\infty\} \cup \{x \in \mathbb{R}^n : \|x\| > R\},$$

   where R ranges over all positive reals. Verify that this is indeed a basis. Then prove that X is a compact Hausdorff space. Not for credit: is X homeomorphic to a space you already know?