Homework 8: MATH 4530

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly.

1. Let X be sequentially compact, and let $f : X \to Y$ be continuous. Must $f(X)$ be sequentially compact?

2. Consider $\mathbb{R}_\ell$, the reals with the lower-limit topology, and its subset $[0,1]_\ell$ endowed with the topology it inherits as a subspace of $\mathbb{R}_\ell$. Is $[0,1]_\ell$ compact? Is it sequentially compact?

3. Let X, Y be compact Hausdorff spaces, and $f : X \to Y$ a continuous bijection. Must f be a homeomorphism?

4. Is $\mathbb{R}_\ell$, the reals with the lower-limit topology, separable? Is it second countable? Is it metrizable? Is it Hausdorff?

5. Let $X_1, X_2, \ldots$ be separable spaces. Show that $\prod_{i=1}^{\infty} X_i$ is separable.