Homework 11: MATH 4530

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly.

1. Show that if A is a retract of the closed disk $\overline{B}^2$, then every continuous map $f : A \to A$ has a fixed point.

2. Show that if A is an invertible 3×3 matrix with non-negative entries, then A has a positive real eigenvalue. Can one replace "invertible" by "not identically zero" in the above statement?

3. Let $\gamma : [0, 1] \to X$ be a loop, with $\gamma(0) = \gamma(1) = x_0$. Show that γ is path homotopic to the constant loop $\alpha(t) = x_0$ for all $t \in [0, 1]$ iff γ is *freely homotopic* to a point, i.e. the map $f : S^1 \to X$, given in complex coordinates by $f(e^{2\pi it}) = \gamma(t)$, is homotopic to a constant map. (This is the notion of *null-homotopic* for loops.)

4. Classify the 26 letters of the Roman alphabet up to homeomorphism. Then classify them up to homotopy equivalence. (If there seem to be several choices for a single letter, using different font etc, choose the one that seems most natural/simple).

5. Show that X is contractible iff it is homotopy equivalent to a one-point space.

6. FInd a space X and a point $x_0 \in X$ such that the inclusion $\{x_0\} \to X$ is a homotopy equivalence, but there is no deformation retract $\{x_0\} \to X$. (See Munkres Exercise 57.9 for a hint).