Homework 6 : MATH 6120

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Gradescope. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly.

1. Prove that if $f(z)$ is holomorphic on the annulus $\{z : R_1 < |z| < R_2\}$, and $M(r) := \sup_{|z|=r} |f(z)|$, then $\log M(e^s)$ is a convex function of s.

2. Recall that a *rhumb line* is a path on the sphere of constant compass bearing (e.g. northwest). Show that any rhumb line has finite length.

3. Evaluate
$$\int_0^{2\pi} \frac{d\theta}{2 - \sin\theta}.$$

4. Show that if $a > 0$, then
$$\int_0^{\infty} \frac{\log x}{x^2 + a^2} dx = \frac{\pi}{2a} \log a.$$