Homework 03: MATH 6210

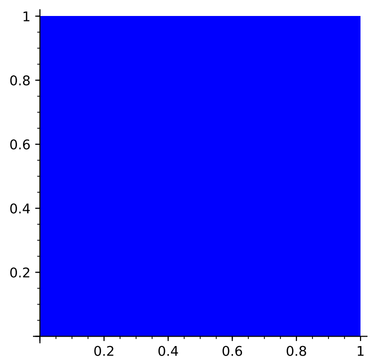
Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Prove the following claim used in lecture: if $E, F \subset \mathbb{R}$ are disjoint compact sets, then

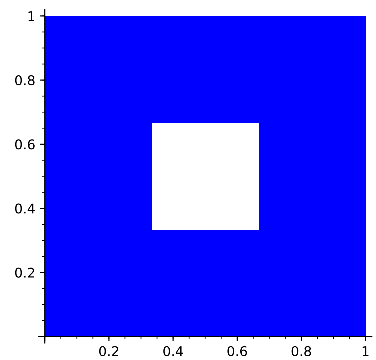
$$d(E, F) > 0.$$

2. Show that every closed subset of $\mathbb{R}$ is a countable intersection of open sets.

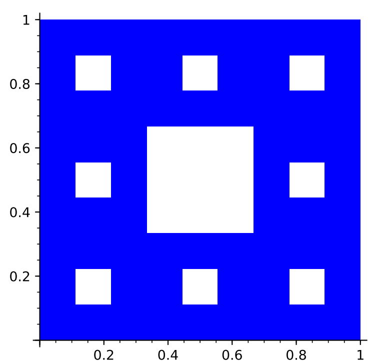
3. We define the *Sierpinski carpet* $\mathcal{S} \subset \mathbb{R}^2$ as the intersection $\bigcap_{n=0}^{\infty} \mathcal{S}_n$, where $\mathcal{S}_n$ is constructed as follows. We start with $\mathcal{S}_0$ a solid square of side length 1, and then obtain $\mathcal{S}_i$ by dividing $\mathcal{S}_{i-1}$ into squares, and removing the middle (1/9)th of each square, as in the images below.



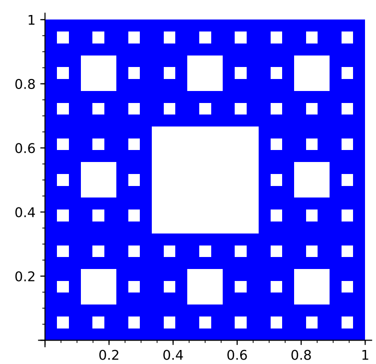
(a) $\mathcal{S}_0$



(b) $\mathcal{S}_1$



(c) $\mathcal{S}_2$



(d) $\mathcal{S}_3$

Show that the Sierpinski carpet $\mathcal{S}$ is measurable and compute its measure.