

## Homework 7 : MATH 6210

**Collaboration Policy** : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  be a measurable function, and  $g : \mathbb{R} \rightarrow \mathbb{R}$  a continuous function. Show that  $g \circ f : \mathbb{R}^n \rightarrow \mathbb{R}$ , i.e. the function  $x \mapsto g(f(x))$ , is measurable.
2. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a measurable function. Show that  $\{(x, y) : 0 \leq y \leq f(x)\}$  is a measurable subset of  $\mathbb{R}^2$ . (Hint: consider the function  $F(x, y) = f(x) - y$ .)
3. Prove that there exists a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  that is *not* measurable.