Homework 9: MATH 6210

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Let $f_n : [0,1] \to \mathbb{R}_{\geq 0}$ be measurable functions. Suppose that the f_n converge to f *uniformly*, where $f : [0,1] \to \mathbb{R}_{\geq 0}$. This means that for any $\epsilon > 0$, there exists $N > 0$ such that if $n \geq N$ then $|f_n(x) - f(x)| < \epsilon$ for all $x \in [0,1]$. Prove that

$$\lim_{n\to\infty} \int f_n = \int f,$$

   (in particular, show that the limit on the left exists).

2. Is it true that if $f, g : \mathbb{R} \to \mathbb{R}_{\geq 0}$ are measurable and Lebesgue integrable, then the product fg must also be Lebesgue integrable?

3. (Worth double points) Prove that if $f : [0,1] \to \mathbb{R}_{\geq 0}$ is *Riemann* integrable, then f is measurable, f is Lebesgue integrable, and the Lebesgue integral of f is equal to the Riemann integral of f.