Syllabus – MAT 6210: Measure Theory and Lebesgue Integration
Fall 2021

Lecture: MW 8:05am - 9:20am

August 29, 2021

Instructor:

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Office hours: Tue 5-6pm, Fri 3-4pm, in office

TA/Grader:

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Office hours: Tuesdays 3-5pm, via Zoom

Prerequisites:

No formal prerequisites, but you should certainly be familiar with writing proofs. Exposure to undergraduate analysis and/or topology would be helpful.

Textbooks:

The Elements of Integration and Lebesgue Measure, by Robert Bartle
Available for free from Cornell libraries: `https://newcatalog.library.cornell.edu/catalog/13053549`
(If you are off-campus, you may need to authenticate with Passkey)

An introduction to measure theory, by Terence Tao (easy to find free pdf online).

Course Description:

From Course Bulletin: "Covers measure theory, integration, and L^p spaces."

The Lebesgue measure on $\mathbb{R}$ is a notion of length that applies even when the set is rather complicated. This generalizes our usual geometric conception of length (and Lebesgue measure in $\mathbb{R}^n$ generalizes notions of area, volume, etc). This can then be used to define the Lebesgue integral, a generalization of the Riemann integral. With this notion, we can integrate more complicated functions, and we get better convergence properties. These properties make the Lebesgue integral crucial in analysis, as well as in fields such probability, ergodic theory, and partial differential equations.

List of topics that we are likely to cover:

- Lebesgue outer measure
- Lebesgue measurable sets
- Properties of Lebesgue measure, notably countable additivity
- Example of a non-Lebesgue measurable set (Vitali set)
- Abstract measure theory, σ-algebras

- The Lebesgue integral
- Convergence theorems (Fatou's Lemma, Monotone Convergence Theorem, Dominated Convergence Theorem)
- Littlewood's three principles of analysis
- Theorems of Egoroff and Lusin
- Connections to probability theory
- Riesz Representation Theorem
- Hausdorff measure and dimension

Homework: There will be a weekly homework assignment - this will be the most important part of the course. You may, in fact are encouraged to, work on the problems with other students. You will write up your solutions by yourself either (i) electronically, or (ii) by hand (legibly), and then scan them (legibly).

Each homework assignment will be due on Tuesday at 10pm.

All assignments will be submitted on Canvas.

Midterm and Final: There will be a take-home midterm assigned in late October. You will have several days to work on this (by yourself). After the submission deadline, I will arrange a time slot with each of you, and we will do a short oral exam covering problems from the take-home part. The final will follow the same procedure, with both take-home and oral parts. The submission deadline will be close to the registrar's scheduled date for the final for this course (still TBA), and the oral exams will be within a day or two of this.

Grading:

- Homework: 40%. The lowest score will be dropped (even if it is zero).
- Take-home part of midterm: 10%
- Oral part of midterm: 15%
- Take-home part of final: 15%
- Oral part of final: 20%