Homework 2 : MATH 6640

Collaboration Policy: You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Canvas. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx or .jpeg, to .pdf.

1. Prove that $O^+(n,1)$ acts transitively on points in H^+.

2. Given n points in $\mathbb{H}^n$, show that there is a totally geodesic subspace of dimension $n-1$ that contains all of them.

3. (Thurston-Levy Ex 2.3.5) Let p be a point on H^+, and let v be a tangent vector to H^+ at p that has length 1 in the hyperbolic metric (i.e. the Minkowski metric). Show that the geodesic through p tangent to v is parametrized with velocity 1 by $p\cosh(t) + v\sinh(t)$. What is the analogous formula for the sphere?

4. Let T be a geodesic triangle on the 2-sphere (with round metric, radius 1). Let the angles of T be α, β, γ (in radians). Show that

$$area(T) = \alpha + \beta + \gamma - \pi.$$

5. Show that any two ideal geodesic triangles in $\mathbb{H}$ are congruent, and that any two triangles with angles $\theta, 0, 0$ are congruent.

6. Prove that there is a hyperbolic octagon with all sides of equal length, and all angles equal to $\pi/4$. Compute the area of this octagon. Is there a hyperbolic quadrilateral with all sides of equal length, and all angles $\pi/2$?