Homework 03: MATH 6640

Collaboration Policy: You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Canvas. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx or .jpeg, to .pdf.

1. Compute the dihedral angle (angle between two adjacent faces) of a regular ideal dodecahedron in $\mathbb{H}^3$.

2. Show that for any positive real numbers α, β, γ with $\alpha + \beta + \gamma < \pi$, there exists a hyperbolic triangle with those angles.

3. Prove that there is a subgroup of $\mathrm{Isom}(\mathbb{H}^n)$ that is (i) generated by finitely many finite order elliptic elements, and (ii) is not discrete.

4. Prove that for any compact, orientable surface of S_g genus $g \geq 2$, there is a hyperbolic surface homeomorphic to S_g.

5. Let ϕ be an orientation-preserving parabolic isometry of $\mathbb{H}^3$, thought of as the upper-half space. Prove that ϕ is conjugate in $\mathrm{Isom}(\mathbb{H}^3)$ to a Euclidean translation.

6. Given two ideal hyperbolic triangles, ABC, $A'B'C'$, we can glue side AB to $A'B'$ by an isometry. In fact, for any such gluing, we can translate by some fixed (hyperbolic) distance d, to get a new gluing by isometries. Suppose we also glue AC to $A'C'$ and BC to $B'C'$; there is similarly a continuous family of such choices for each glued pair. Topologically, all these surfaces are a thrice-punctured sphere. Determine which of the gluings give a *complete* hyperbolic surface.