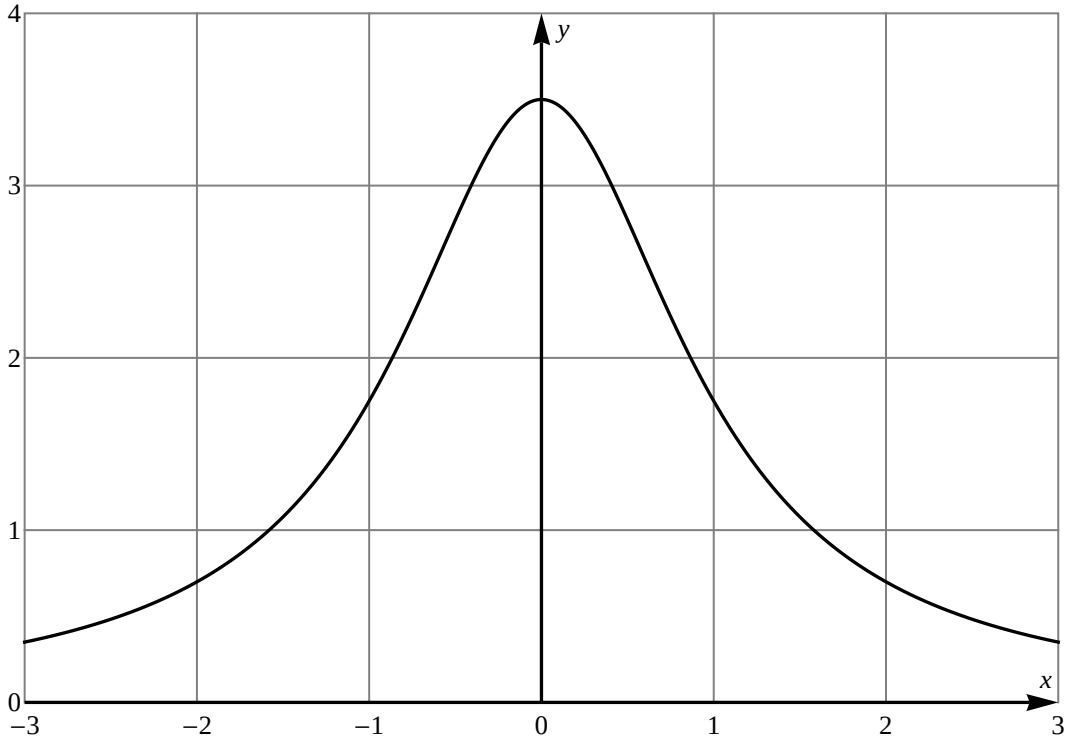
Math 141

Name:_____

Homework 2

1. (a) Use a straightedge to draw tangent lines to the following graph at $x = -3, -2, -1, 0, 1, 2$, and 3.

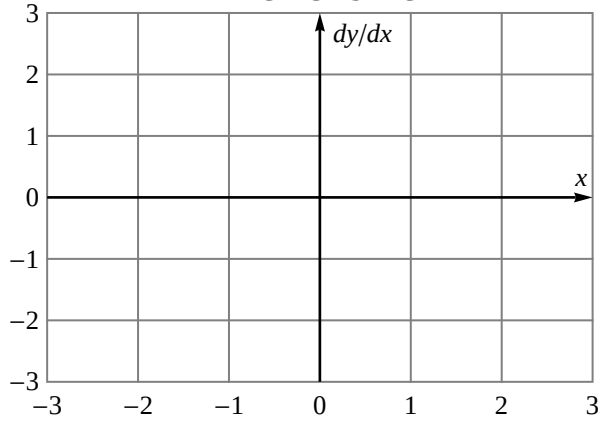


(b) Use your tangent lines to estimate the values of $\frac{dy}{dx}$ for $x = -3, -2, -1, 0, 1, 2$, and 3. Make a table showing your answers.

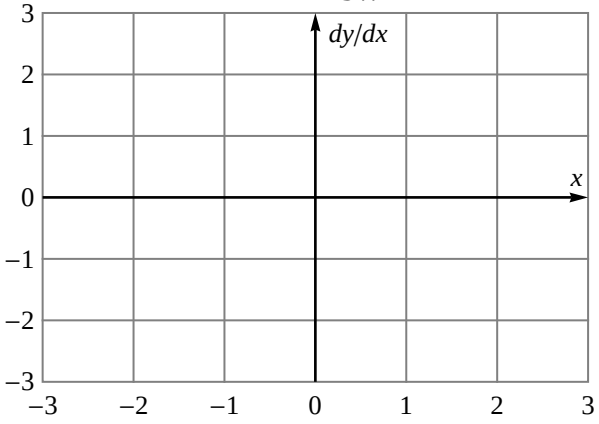
(c) The graph on the previous page was obtained from the equation $y = \dfrac{3.5}{1+x^2}$. Use this formula to obtain more accurate estimates for $\dfrac{dy}{dx}$ at $x = -3$, -2, -1, 0, 1, 2, and 3. Your answers must be correct to two decimal places.

(d) Use your answers to part (c) to sketch a rough graph of $\dfrac{dy}{dx}$ as a function of x.

PRACTICE SPACE



FINAL ANSWER



2. Water is draining out of the bottom of a 5000-gallon tank. The volume V of water (in gallons) remaining in the tank after t minutes is given by the following formula:

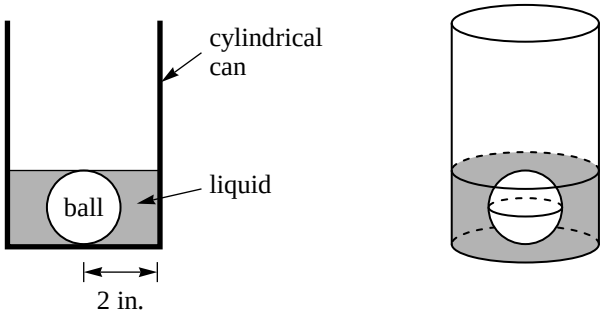
$$V = 5000\left(1 - \frac{t}{40}\right)^2$$

(a) How long does it take for the tank to drain completely? Explain.

(b) Find the average rate at which water drains from the tank during the first 30 minutes. Make sure your answer includes the proper units.

(c) Estimate the rate at which water is draining from the tank at $t = 30$ minutes. Make sure your answer includes the proper units.

3. A heavy metal ball is placed into a tin can, and then liquid is added until the top of the ball is just barely covered:



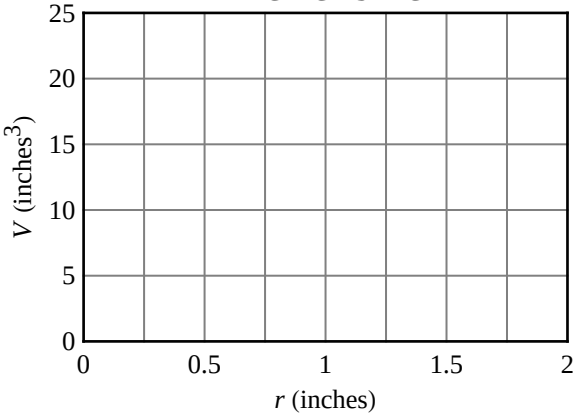
The can is a cylinder with a radius of 2 inches, and the ball is a sphere of radius r.

(a) Compute the volume of liquid in the can when r is 0.5 inches.

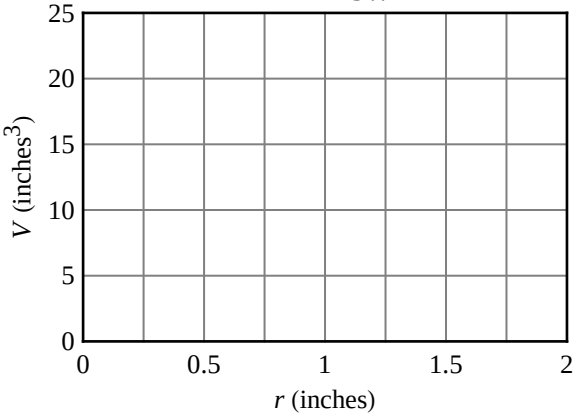
(b) Find a formula for the volume V of liquid in the can as a function of r. Make sure that your formula is consistent with your answer to part (a).

(c) Use the following axes to draw a careful graph of V as a function of r. Feel free to use a graphing calculator or computer to help you with this part.

PRACTICE SPACE



FINAL ANSWER



(d) Give an intuitive explanation for the shape of the curve in part (c). Why is the volume increasing between $r = 0.5$ inches and $r = 1$ inch? Why is the volume decreasing between $r = 1.5$ inches and $r = 2$ inches?

(e) Use a graphing calculator or computer to find the value of r that requires the maximum amount of liquid. Your answer should be correct to within 0.01.