Math 141

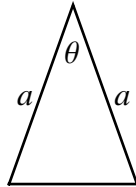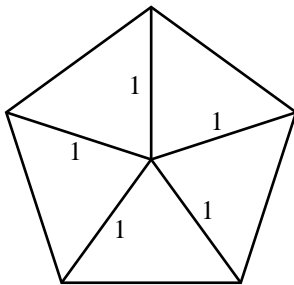# Homework 3

1. Consider the following formula for the area of an isosceles triangle:
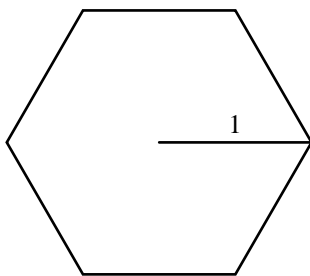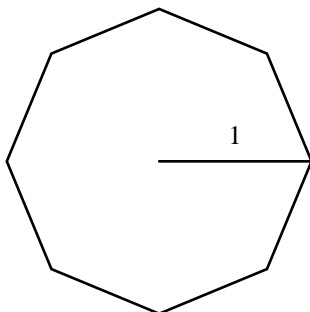

$$\text{Area} = \frac{1}{2}a^2 \sin\theta$$

(a) A **unit pentagon** is a regular pentagon with a radius of 1, as shown in the following figure. Use the area formula above to find the area of a unit pentagon.

(b) Find the area of a unit hexagon. Express your answer as a decimal.

(c) Find the area of a unit octagon. Express your answer as a decimal.

Let $f(n)$ be the area of a unit polygon with n sides.

(d) Find a formula for $f(n)$. Make sure to check that your formula is consistent with your answers to parts (a), (b), and (c).

(e) Make a table of values for f showing $f(10)$, $f(100)$, $f(1000)$, and $f(10,000)$. Based on your table, what is the value of $\lim_{n\to\infty} f(n)$?

(f) Give a brief geometric explanation for your answer to part (e).

2. Consider the following sum:
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

The first term is $1/2$, and each successive term is half as big as the previous term. For example, $1/4$ is half of $1/2$, and $1/8$ is half of $1/4$. The sum goes on forever, with infinitely many terms.

(a) List the denominators of the first ten terms.

(b) Let $g(n)$ be the result of adding together the first n terms. For example,

$$g(1) = \frac{1}{2}, \qquad g(2) = \frac{1}{2} + \frac{1}{4}, \qquad \text{and} \qquad g(3) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}.$$

Make a table showing the value of $g(n)$ for $n = 1, 2, \ldots, 10$. Express your answers as decimals.

(c) Based on your answer to part (b), what would the result be of adding together *all* of the terms? Explain.

3. Consider the function $f(x) = x^3$. In this problem, we will use a limit to find the value of $f'(2)$.

(a) Find the average slope of f between $x = 2$ and $x = 2.01$.

(b) Find a formula for the average slope of f between $x = 2$ and $x = 2 + h$. Make sure that your formula is consistent with your answer to part (a).

(c) Use your answer to part (b) to write a limit whose value is $f'(2)$.

(d) Make a table of data for the limit in part (c), showing the slope for $h = 0.1$, $h = 0.01$, $h = 0.001$, and $h = 0.0001$. Based on your table, what is the value of $f'(2)$?

(e) Instead of using a table, it is possible to evaluate the limit in part (c) using algebraic reasoning. To begin, use algebra to simplify your formula from part (b) as much as possible.

(f) Now, explain how you can tell the value of the limit from the simplified form you found in part (e).