Math 141

Name:_____

Homework 4

1. In the theory of electrical circuits, ***Ohm's law*** describes the relationship between the voltage V across a resistor, the electrical current I passing through the resistor, and a quantity R known as the resistance. The law can be written as follows:

$$V = IR$$

Usually voltage is measured in volts, current is measured in amperes (amps), and resistance is measured in ohms, where 1 ohm $=$ 1 volt/amp. In a circuit with variable resistance, the quantities V, I, and R might all depend on time.

(a) Take the derivative of Ohm's law to find an equation relating $\frac{dV}{dt}$, $\frac{dI}{dt}$, and $\frac{dR}{dt}$.

(b) Suppose that the current is increasing at a rate of 0.3 amps/sec, while the resistance is holding steady at 4 ohms. How quickly is the voltage across the resistor increasing?

Now suppose that the voltage across the resistor is held constant at 20 volts, while the resistance is steadily increased at a rate of 0.4 ohms/sec.

(c) What is the current through the resistor when the resistance reaches 10 ohms?

(d) At what rate is the current changing at that time? Is it increasing or decreasing?

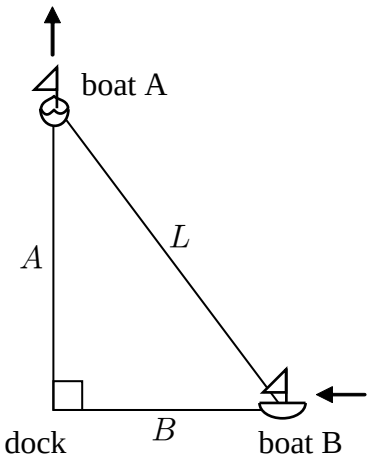
2. A spherical snowball with a radius of 2 inches is melting. The volume V and radius r of the snowball are related by the formula:

$$V = \frac{4}{3}\pi r^3$$

(a) Find a formula for $\frac{dV}{dt}$ in terms of r and $\frac{dr}{dt}$.

(b) Because the snowball is melting, the radius of the snowball is decreasing at a rate of 0.7 in./hour. How quickly is the volume of the snowball decreasing?

3. Boat A is sailing north away from a dock, while boat B is sailing west towards the same dock:



In the picture above, the variable A represents the distance from boat A to the dock, the variable B represents the distance from boat B to the dock, and the variable L represents the distance between the two boats. By the Pythagorean theorem:

$$A^2 + B^2 = L^2$$

(a) Take the derivative of the above equation to find an equation involving $\frac{dA}{dt}$, $\frac{dB}{dt}$, and $\frac{dL}{dt}$.

(b) Is $\frac{dA}{dt}$ positive or negative? What about $\frac{dB}{dt}$? Explain.

Suppose that boat A is 30 km north of the dock, and is sailing north at a rate of 10 km/hour. Meanwhile, boat B is 40 km east of the dock, and is sailing west at a rate of 15 km/hour.

(c) What is the present distance between the two boats?

(d) What is $\frac{dA}{dt}$? What is $\frac{dB}{dt}$?

(e) How quickly is the distance between the boats changing? Is the distance increasing or decreasing?