Math 141

Name:_____

Homework 5

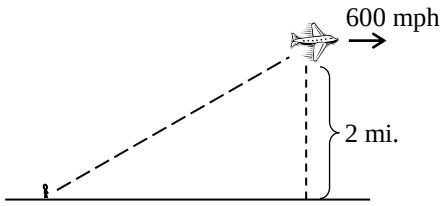
1. When expanding adiabatically (i.e. without gaining or losing heat), the pressure P and volume V of a sample of argon gas are related by the formula

$$PV^{5/3} = \text{constant}.$$

   (a) Take the derivative of the above equation to find a formula relating P, V, $\frac{dP}{dt}$ and $\frac{dV}{dt}$.

   (b) Suppose the volume of the sample is increasing at a rate of $0.4\ \text{L/s}$. How quickly is the pressure decreasing when the volume is $2.7\ \text{L}$ and the pressure is $80\ \text{kPa}$?

2. An airplane flies directly over an observer standing on the ground. The following picture shows the position of the plane a short while later:



The plane maintains a constant altitude of 2 miles, and a constant forward speed of 600 mph.

(a) What is the distance from the observer to the plane 30 seconds after the plane flies overhead?

(b) How quickly is the distance from the observer to the plane increasing at that time?

3. The radius of a circle is increasing at a rate of $0.3\ \text{cm/s}$. How quickly is the area of the circle increasing when the area is $120\ \text{cm}^2$?

4. A positively charged particle is flying in the vicinity of a charged conductor. The electric potential energy of the particle is given by the formula

$$E = k_e \frac{qQ}{r},$$

where q is the charge of the particle, Q is the charge on the conductor, r is the distance between them, and $k_e = 0.90\ \text{cm} \cdot \text{J}/\mu\text{C}^2$.

(a) Assuming q and Q are constant, find a formula for $\dfrac{dE}{dt}$ in terms of q, Q, k_e, r and $\dfrac{dr}{dt}$.

(b) At a certain instant, a particle with a charge of 1.5 μC is 20 cm away from a conductor, and is flying directly towards the conductor at a rate of 2.0 cm/s. Given that the conductor has a charge of 4.0 μC, how quickly is the electrical potential energy of the particle increasing?

5. Water is draining out of the bottom of a 5000-gallon tank. The volume V of water (in gallons) remaining in the tank after t minutes is given by the following formula:

$$V = 5000\left(1 - \frac{t}{40}\right)^2$$

(a) Find a formula for $\dfrac{dV}{dt}$ in terms of t.

(b) How quickly is water initially draining from the tank?

(c) How quickly is water draining from the tank at $t = 30$ minutes?