Math 141

Name:_____

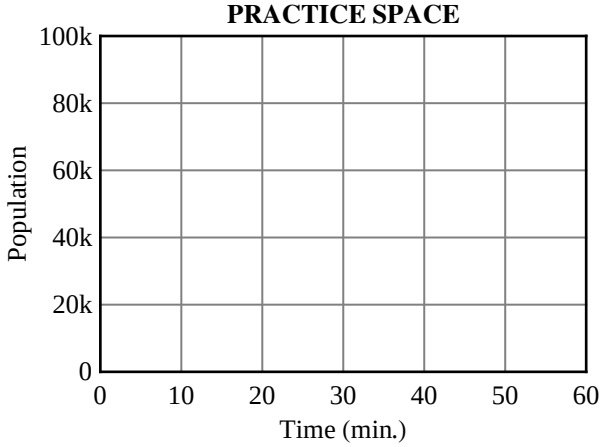
Homework 8

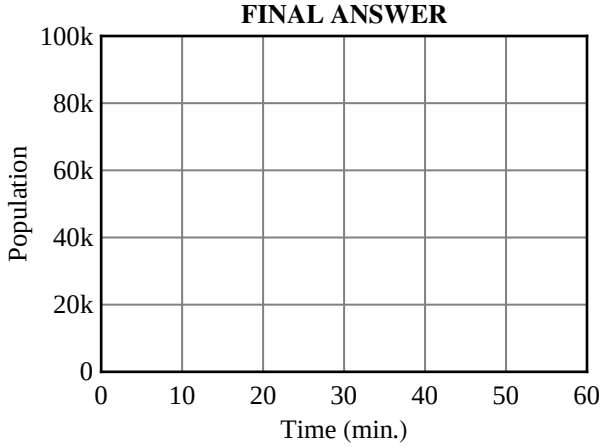
1. At the beginning of a biology experiment, a culture of *E. coli* bacteria has a population of 6,000. Ten minutes later, the population has increased to 9,300.

   (a) What was the percentage increase in the population of *E. coli* over the first ten minutes?

   (b) Assuming exponential growth, make a table showing the population of *E. coli* after 20 minutes, 30 minutes, 40 minutes, 50 minutes, and 60 minutes.

   (c) Use the following axes to sketch a graph of the *E. coli* population during the first hour of the experiment.

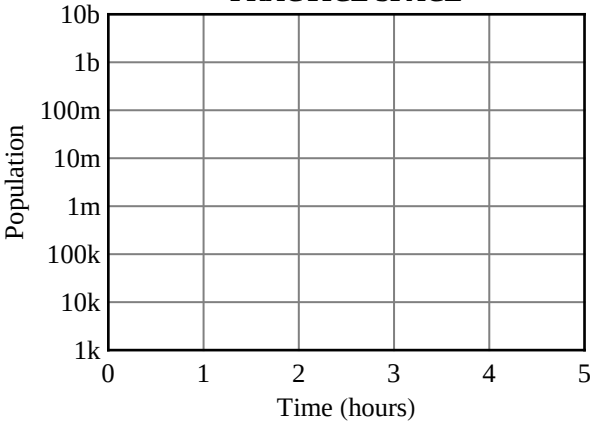




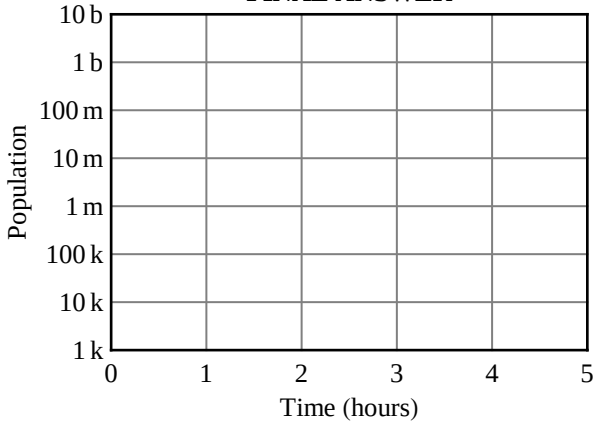
(d) Suppose the bacteria continue to grow exponentially for five hours. Make a table showing the population of *E. coli* at the end of each hour.

(e) Use the following axes to sketch a logarithmic plot of the *E. coli* population over the course of five hours.

PRACTICE SPACE



FINAL ANSWER



(f) Assuming exponential growth, the population P should obey the formula

$$P = be^{kt}$$

where t is the time in minutes, and b and k are constants. What are the values of b and k?

(g) The ***generation time*** of a bacterial culture is length of time required for the population to double. What is the generation time of this *E. coli* culture? Your answer must be correct to two decimal places.

(h) Find a formula for $\frac{dP}{dt}$ as a function of t.

(i) Based on your answer to part (h), how quickly was the population increasing at the very beginning of the experiment?

2. As part of a chemistry experiment, 0.250 moles of butyl chloride (C_4H_9Cl) are dissolved in water. The butyl chloride reacts with the water, producing butyl alcohol and hydrochloric acid. Initially, this reaction consumes butyl chloride at a rate of 0.030 moles/min.

   Let t be the time in minutes, and let n be the number of moles of butyl chloride remaining.

   (a) Assuming that n decays exponentially, find a formula for n in terms of t.

   (b) How much butyl chloride will remain after 10 minutes?

   (c) How quickly is the butyl chloride being consumed at this time?

   (d) How long will it take for 95% of the butyl chloride to be consumed?

3. In astronomy, the ***apparent magnitude*** is a logarithmic measure of the brightness of a star as seen by an observer on Earth. The apparent magnitude M of a star is related to the brightness B of the observed light by the formula

$$M = -1.09 \ln\left(\frac{B}{B_0}\right)$$

where $B_0 = 2.13 \times 10^{-6}$ lux. Note that brighter stars have *smaller* apparent magnitudes.

(a) As seen from Earth, the star Polaris (the North Star) has a brightness of 3.5×10^{-7} lux. What is the apparent magnitude of Polaris?

(b) The brightest star in the night sky is Sirius, with an apparent magnitude of -1.47. What is the brightness of the light that the Earth receives from this star?

(c) To an observer on Earth, the sun is approximately 12 billion times as bright as Sirius. What is the apparent magnitude of the sun?

(d) Find a formula relating $\frac{dM}{dt}$ and $\frac{dB}{dt}$.

(e) The apparent magnitude of the variable star Delta Cephei oscillates regularly over the course of several days. At a certain time, the apparent magnitude of Delta Cephei is 3.90, and the magnitude is decreasing at a rate of $0.010/\text{hour}$. How quickly is the brightness of the star increasing?