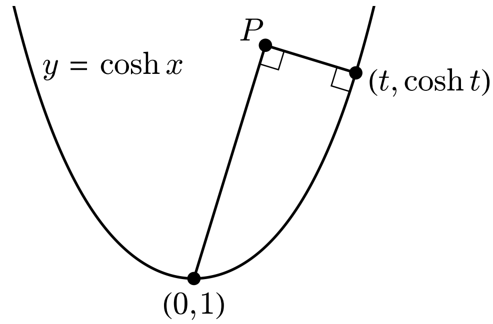
Math 352

Name:________________________________

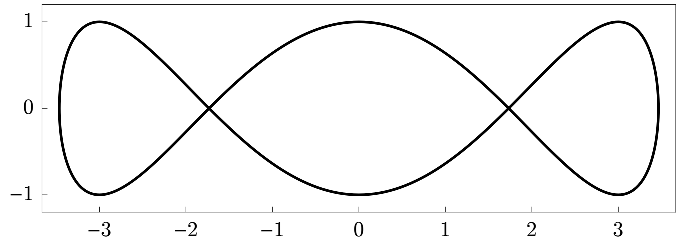
Exam 1

1. [**8 points**] Find a unit-speed parametrization for the curve $r = e^{2\theta}$.

2. [**10 points**] In the following figure, find the coordinates of the point P in terms of t.



3. [**36 points**] The following picture shows the curve $\vec{x}(t) = \big(2\sqrt{3}\cos t,\ \sin 3t\big)$.

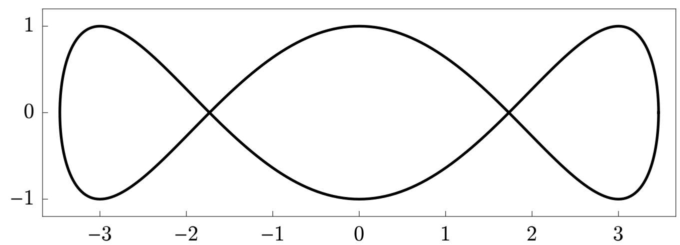


(a) [**4 pts**] Compute the unit tangent vector to this curve at $t = \pi/3$.

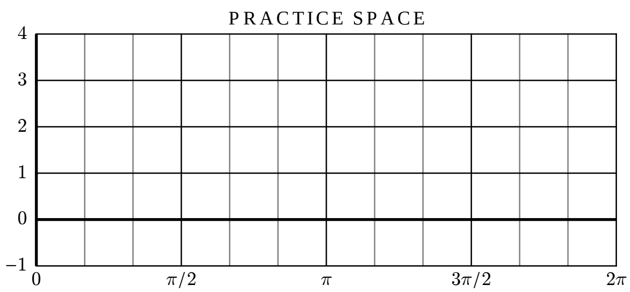
(b) [**8 pts**] Find the Cartesian equation of the osculating circle to this curve at the point $(0, -1)$.

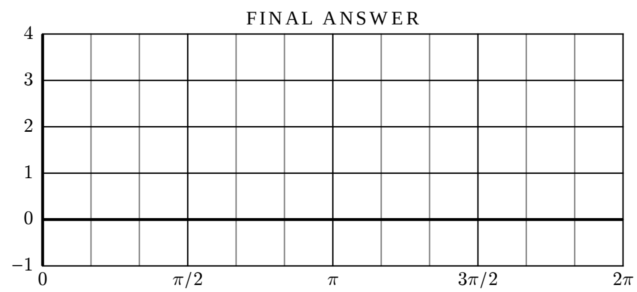
(c) [**4 pts**] Based on the given picture, how many vertices does the curve have? Draw points on the picture showing the approximate positions of these vertices.

The following picture shows the curve $\vec{x}(t) = \left(2\sqrt{3}\cos t,\, \sin 3t\right)$.



(d) **[8 pts]** Use the picture to make a rough sketch of the curvature function $\kappa_g(t)$.





(e) **[4 pts]** Evaluate $\displaystyle\int_{\mathcal{C}} \kappa_g(s)\, ds$.

(f) **[4 pts]** Determine the winding number of this curve around each of the following points: $(3, 0)$, $(0, 0)$, and $(-3, 0)$.

(g) **[4 pts]** The value of $\displaystyle\int_{\mathcal{C}} x\, dy$ is (choose one):

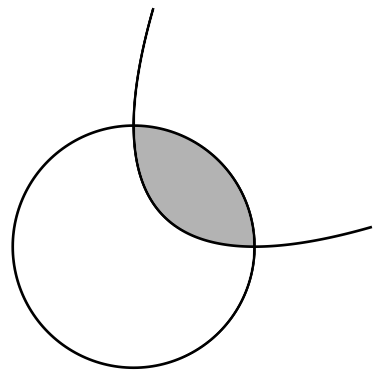
(a) less than -5 | (b) between -5 and -1 | (c) between -1 and 1

(d) between 1 and 3 | (e) between 3 and 5 | (f) greater than 5

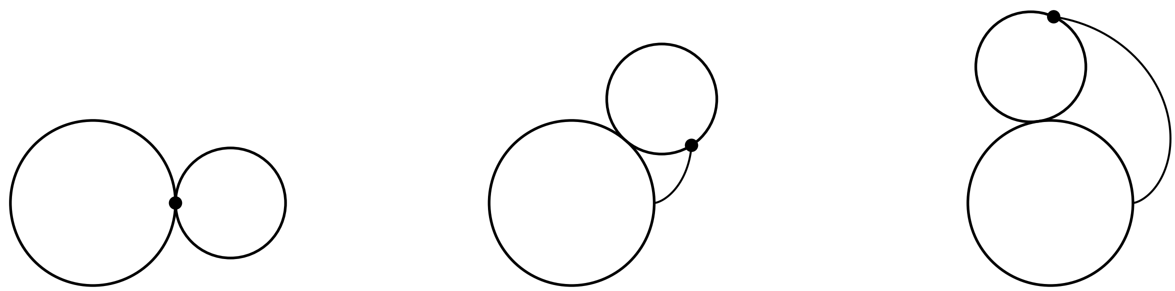
4. [**6 points**] Evaluate $\displaystyle\int_{\mathcal{C}} 3x^2\cos(y^2)\,dx \;-\; 2x^3y\sin(y^2)\,dy$, where $\mathcal{C}$ is any curve from the point $\left(1, \sqrt{\pi}\right)$ to the point $(2, 0)$.

5. [**8 points**] Suppose that a regular parametric curve $\vec{x}(t)$ has curvature $\kappa_g(t) = 3t^2$ and speed $s'(t) = 2t^3 + 1$. Given that $\vec{x}\,'(0) = (0, 1)$, find a formula for the velocity $\vec{x}\,'(t)$ as a function of t.

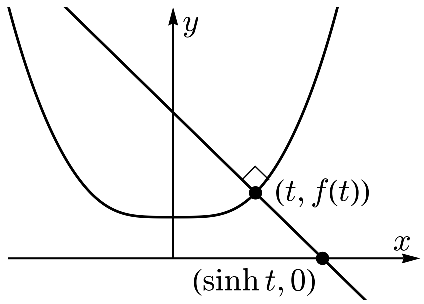
6. [**12 points**] Use Green's Theorem to evaluate $\iint_{\mathcal{R}} x\,dA$, where $\mathcal{R}$ is the region bounded by the circle $x^2 + y^2 = 1$ and the curve $\vec{x}(t) = \left(t^2, (1-t)^2\right)$ shown in the following figure.



7. [**14 points**] A circle of radius $2/3$ is rolling counterclockwise around the unit circle $x^2+y^2=1$. A point P lies on the perimeter of the rolling circle, with initial coordinates $(1,0)$. Find parametric equations for the curve produced by tracing the path of P.



8. [**6 points**] For a certain differentiable function $f(x)$, the normal line to the graph at each point $\big(t, f(t)\big)$ passes through the x-axis at the point $(\sinh t, 0)$.



Given that $f(0) = 1$, find a formula for f.