

Quiz 4

1. [4 points] Use the definition of the derivative to find $f'(2)$ if $f(x) = 5x^2$.

Solution #1

$$\begin{aligned}
 f'(2) &= \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} \\
 &= \lim_{x \rightarrow 2} \frac{5x^2 - 20}{x - 2}
 \end{aligned}
 \rightarrow
 \begin{aligned}
 &= \lim_{x \rightarrow 2} \frac{5(x^2 - 4)}{x - 2} \\
 &= \lim_{x \rightarrow 2} \frac{5(x - 2)(x + 2)}{x - 2} \\
 &= \lim_{x \rightarrow 2} 5(x + 2) = \boxed{20}
 \end{aligned}$$

Solution #2

$$\begin{aligned}
 f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5(2+h)^2 - 20}{h}
 \end{aligned}
 \rightarrow
 \begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{5(h^2 + 4h + 4) - 20}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5h^2 + 20h}{h} \\
 &= \lim_{h \rightarrow 0} 5h + 20 = \boxed{20}
 \end{aligned}$$

2. [6 points] Find the derivatives of the following functions.

(a) $\frac{x^4}{2} + 4 \sin x - e^x = \frac{1}{2} x^4 + 4 \sin x - e^x$

So $\boxed{2x^3 + 4 \cos x - e^x}$

(b) $2 + \sqrt{x} \cos x$

$$\boxed{\frac{1}{2\sqrt{x}} \cos x + \sqrt{x} (-\sin x)}$$