The Derivative as a Function

Study Guide

Problems listed in parentheses are for extra practice.

1. Graphs of the Derivative

You should make sure that you feel comfortable with the relationship between the graph of $f(x)$ and the graph of $f'(x)$. Remember that:

- Straight segments in the graph of $f(x)$ become horizontal (constant) portions of the graph of $f'(x)$.
- Bends (or corners) on the graph of $f(x)$ become jumps on the graph of $f'(x)$, and $f'(x)$ isn't defined at the jump.
- Vertical tangent lines and cusps on the graph of $f(x)$ become vertical asymptotes on the graph of $f'(x)$.

Problems: Section 3.2 # 27–30, 31, (33), 35,

Answers: 27. (b)   28. (a)   29. (d)   30. (c)

2. One-Sided Derivatives

A function has a bend (or corner) at $x = a$ if the left-hand derivative and right-hand derivative have different values at $x = a$. In this case, the function is not differentiable at $x = a$.

Problems: Section 3.2 # 37, (39), 41, (43)

3. Differentiability

A function will fail to be differentiable at $x = a$ if:

- It is not continuous at $x = a$, or
- It has a bend (or corner) at $x = a$. These correspond to points where the left-hand and right-hand derivatives are different.
- It has a vertical tangent line or cusp at $x = a$. These correspond to points where the one-sided derivatives approach ∞ or $-\infty$.

Problems: Section 3.2 # 45, (46), 47, 48, (49), (50)

Answers:

46. continuous and differentiable everywhere

48. continuous but not differentiable at $x = -1$, neither continuous nor differentiable at $x = 0$ or $x = 2$

50. continuous but not differentiable at $x = -2$ and $x = 2$