Derivatives and Graphs

Study Guide

1. Linearizations

The **linearization** of a function $f(x)$ at a point $x = a$ is another name for the tangent line at $x = a$. The formula for the linearization is

$$L(x) = f(a) + f'(a)\,(x - a).$$

You can use $L(x)$ as an approximation for $f(x)$ near $x = a$. This is called a **linear approximation**.

Problems: Section 3.11 # (1), (3), (5)

2. Critical Points and Extreme Values

A point c is called a **critical point** for a function $f(x)$ if either

(a) $f'(c) = 0$, or

(b) $f'(c)$ does not exist.

If $f(x)$ is continuous on an interval $[a, b]$, then $f(x)$ must have an absolute maximum and absolute minimum value on this interval. These must occur at either critical points or endpoints.

Problems: Section 4.1 # 1, 3, (5), (7), (9), 23, 35, 49, (53)

3. First Derivative Test

A function is increasing when $f'(x)$ is positive, and decreasing when $f'(x)$ is negative.

The first derivative text lets you identify critical points for a continuous function f:

- A critical point at which f switches from increasing to decreasing is a local max.
- A critical point at which f switches from decreasing to increasing is a local max.
- Any other critical point is neither.

Problems: Section 4.3 # 1, 3, 11, 21, (27), (41), (43), 67