

Exponents and Logarithms

Study Guide

1. Exponent Laws

You should be familiar with exponents and the rules for manipulating them, including:

- $(ab)^x = a^x b^x$ and $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$.
- $a^x a^y = a^{x+y}$ and $\frac{a^x}{a^y} = a^{x-y}$.
- $(a^x)^y = a^{xy}$.

You should also know the meaning of negative and fractional exponents:

- $a^{-x} = \frac{1}{a^x}$.
- $a^{1/n} = \sqrt[n]{a}$.

Problems: Section 1.5 # 11, 13, 15, 17, 19

2. The Natural Logarithm

The natural logarithm $\ln x$ is the same as the logarithm base e , where $e = 2.71828\dots$ is a certain irrational constant. There will be an explanation later for why this number e is so important. The natural logarithm is defined by the equations

$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x.$$

When $x = 0$ the first equation says that $\ln(1) = 0$, and when $x = 1$ the first equation says that $\ln(e) = 1$.

The main rules for logarithms follow from the rules for exponents:

- (a) $\ln(xy) = \ln x + \ln y$ and $\ln(x/y) = \ln x - \ln y$.
- (b) $\ln(x^k) = k \ln x$.

Rule (b) has special cases when $k = -1$ and when $k = 1/n$:

$$\ln(1/x) = -\ln x, \quad \ln(\sqrt[n]{x}) = \frac{1}{n} \ln x.$$

Problems: Section 1.6 # 41, (43), 45, 47