Functions

Study Guide

1. Domain and Range

The **domain** of a function f is the set of possible inputs or x-values. For example:

- The domain of $f(x) = x^2$ is $(-\infty, \infty)$.
- The domain of $f(x) = \sqrt{x}$ is $[0, \infty)$.

The **range** of a function f is the set of possible outputs or y-values. For example:

- The range of $f(x) = x^2$ is $[0, \infty)$.
- The range of $f(x) = x^3$ is $(-\infty, \infty)$.

Problems: Section 1.1 # 1, 3

2. Piecewise Functions

A function like

$$f(x) = \begin{cases} x^2 & \text{if } x < 3 \\ x + 1 & \text{if } x \geq 3 \end{cases}$$

is a **piecewise function**. It uses the formula x^2 when the input is less than 3, and uses the formula $x + 1$ when the input is 3 or greater.

Problems: Section 1.1 # 25, 31

3. Even and Odd Functions

A function f is **even** if $f(-x) = f(x)$, and is **odd** if $f(-x) = -f(x)$. Many functions are neither even nor odd.

Problems: Section 1.1 # 47–61 odd

4. Compositions of Functions

If f and g are functions, their **composition** $f \circ g$ is the function whose formula is $f(g(x))$. For example, if $f(x) = x^2$ and $g(x) = x + 5$, then

$$f(g(x)) = f(x + 5) = (x + 5)^2$$

so $f \circ g$ is the function $(x + 5)^2$.

Problems: Section 1.2 # 7, 9, 11, 13

5. Shifting and Scaling

If you know the graph of a function $f(x)$, then:

- The graph of $f(x-5)$ is the graph of $f(x)$ shifted right 5 units.
- The graph of $f(2x)$ is the graph of $f(x)$ scaled horizontally by a factor of $1/2$.
- The graph of $f(x)+5$ is the graph of $f(x)$ shifted up five units.
- The graph of $2\,f(x)$ is the graph of $f(x)$ scaled vertically by a factor of 2.

Note that messing with the input of a function changes the graph horizontally, while messing with the output changes the graph vertically. Also, messing with the input always does the opposite of what you might expect.

Problems: Section 1.2 # 23, 25, 29, 31, 37, 39, 41, 59, 63, 71

6. Inverse Functions

Every one-to-one function f has an **inverse** f^{-1}, which represents the opposite operation. For example:

- If $f(x) = x + 5$ then $f^{-1}(x) = x - 5$.
- If $f(x) = 2x$ then $f^{-1}(x) = x/2$.

The graph of f^{-1} is obtained from the graph of f by reflecting across the line $y = x$.

You can find a formula for f^{-1} by starting with the equation $y = f(x)$, interchanging x and y, and then solving for y. For example, if $f(x) = 3x + 1$, then we start with the equation

$$y = 3x + 1.$$

Interchanging x and y gives

$$x = 3y + 1$$

and solving this for y yields

$$y = (x - 1)/3$$

so the inverse is $f^{-1}(x) = (x - 1)/3$.

Problems: Section 1.6 # 11, 13, 19, 21, 23, 25, 27