Inverse Trig and Related Rates

Study Guide

1. Inverse Trig Functions

Inverse trigonometric functions are the inverses of trigonometric functions. For example, $\sin^{-1}(1/2)$ is the angle whose sine is $1/2$, namely $\pi/6$. This is also sometimes written $\arcsin(1/2)$.

Since different angles can have the same sine, cosine, or tangent, we restrict the inverse trig functions to only give values in a certain range. In particular:

- $\sin^{-1}(x)$ is always between $-\pi/2$ and $\pi/2$.
- $\cos^{-1}(x)$ is always between 0 and π.
- $\tan^{-1}(x)$ is always between $-\pi/2$ and $\pi/2$.

The derivatives of these three inverse trig functions are as follows:

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\left[\cos^{-1} x\right] = -\frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}.$$

Problems: 1–14. I recommend solving all of these problems without a calculator.

2. Related Rates

These problems (excluding # 15–18) have the following steps:

(a) Write down an equation that describes the given situation.

(b) Use the chain rule to take the derivative of the given equation with respect to t.

(c) Plug in the given information and solve for the desired quantity.

Problems: 15–24.

Exercises: Inverse Trig and Related Rates

1–10 ■ Compute the values of the following inverse trig functions. Do not use a calculator.

1. $\sin^{-1}(1)$

2. $\arccos\left(\frac{1}{2}\right)$

3. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

4. $\arctan(0)$

5. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

6. $\cos^{-1}(0)$

7. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

8. $\tan^{-1}(-1)$

9. $\sin^{-1}(-1)$

10. $\arctan\left(\sqrt{3}\right)$

11–12 ■ Find the derivative of the given function.

11. $\tan^{-1}\left(\sqrt{x}\right)$

12. $\arcsin\left(e^{3x}\right)$

13. Find the equation of the tangent line to the curve $y = \tan^{-1}(x)$ at $x = 1$.

14. Find the equation of the tangent line to the curve $y = \arcsin(2x)$ at $x = 1/4$.

15–18 ■ Take the derivative of the given equation with respect to t.

15. $A = \pi r^2$

16. $a^2 + b^2 = c^2$

17. $y = x\tan\theta$

18. $V = \frac{1}{3}\pi r^2 h$

19. The radius of a circle is increasing at a rate of 5 cm/min. How quickly is the area of the circle increasing when the radius is 30 cm?

20. The side length of a square is increasing at a rate of 3 cm/sec. How quickly is the area of the square increasing when the area is 100 cm^2?

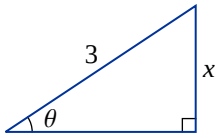
21. The length of a rectangle is increasing at a rate of 5 feet/min, while the width is decreasing at a rate of 3 feet/min. How quickly is the area of the rectangle changing when the length is 20 feet and the width is 10 feet? Is the area increasing or decreasing?

22. The magnetic flux Φ through a loop of wire depends on the magnetic field B and the area A according to the formula $\Phi = AB$.

(a) Suppose that the area of a loop is constant at 10 cm^2, while the magnetic field is increasing at a rate of 0.30 Tesla/sec. How quickly is the flux through the loop increasing?

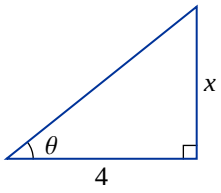
(b) Suppose instead that the area is increasing at a rate of 2.0 cm^2/sec, while the magnetic field is increasing at a rate of 0.15 Tesla/sec. How quickly is the flux increasing when the area is 10 cm^2 and the magnetic field is 0.80 Tesla?

23. In the following triangle, the length x is increasing at a rate of 0.5 units/sec.



How quickly is the angle θ increasing when $\theta = \pi/3$?

24. In the following triangle, the angle θ is increasing at a rate of 0.1 rad/sec.



(a) How quickly is x increasing when $\theta = \pi/4$?

(b) How quickly is x increasing when $x = 3$?

Answers to the Exercises

1. $\pi/2$ **2.** $\pi/3$ **3.** $\pi/3$ **4.** 0 **5.** $-\pi/4$ **6.** $\pi/2$ **7.** $5\pi/6$ **8.** $-\pi/4$ **9.** $-\pi/2$ **10.** $\pi/3$

11. $\dfrac{1}{2\sqrt{x}(1+x)}$ **12.** $\dfrac{3e^{3x}}{\sqrt{1-e^{6x}}}$ **13.** $y = \dfrac{\pi}{4} + \dfrac{1}{2}(x-1)$ **14.** $y = \dfrac{\pi}{6} + \dfrac{4}{\sqrt{3}}\left(x - \dfrac{1}{4}\right)$

15. $\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$ **16.** $2a\dfrac{da}{dt} + 2b\dfrac{db}{dt} = 2c\dfrac{dc}{dt}$ **17.** $\dfrac{dy}{dt} = \tan(\theta)\,\dfrac{dx}{dt} + x\sec^2(\theta)\,\dfrac{d\theta}{dt}$

18. $\dfrac{dV}{dt} = \dfrac{\pi}{3}\left(2rh\dfrac{dr}{dt} + r^2\dfrac{dh}{dt}\right)$ **19.** 300π cm^2/min **20.** 60 cm^2/sec

21. decreasing at 10 feet^2/min **22.** (a) 3.0 Tesla $\cdot$ cm^2/sec (b) 3.1 Tesla $\cdot$ cm^2/sec

23. 1/3 rad/sec **24.** (a) 0.8 units/sec (b) 0.625 units/sec