Limits
Study Guide

Problems listed in parentheses are for extra practice.

1. Limits by Factoring
Sometimes you can find a limit by factoring the numerator and/or denominator. For example:
\[
\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x - 3)(x + 3)}{x - 3} = \lim_{x \to 3} x + 3 = 6.
\]

Problems: Section 2.2 # 23, 25, (27), (29)

2. Conjugate Expressions
When taking the limit of an expression whose numerator or denominator includes a square root, it often helps to multiply through by the conjugate of the radical expression. For example:
\[
\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3} = \lim_{x \to 9} \frac{x - 9}{(x - 9)(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{1}{\sqrt{x} + 3} = \frac{1}{6}
\]

Problems: Section 2.2 # 37, 39, (41)

3. Limit Laws
You should understand the limit laws listed in Theorem 1 (pg. 66) and used in Example 5.
Problems: Section 2.2 # 51, 53, (55)

4. Sandwich Theorem
The sandwich theorem says that if \( f, g, \) and \( h \) are three functions and
\[
f(x) \leq g(x) \leq h(x)
\]
for all values of \( x \) close to \( a \), and
\[
\lim_{x \to a} f(x) = L \quad \text{and} \quad \lim_{x \to a} h(x) = L
\]
for some number \( L \), then we can conclude that
\[
\lim_{x \to a} g(x) = L.
\]
Problems: Section 2.2 # 63, (65a)
5. One-Sided Limits

The expressions
\[ \lim_{x \to a^-} f(x) \quad \text{and} \quad \lim_{x \to a^+} f(x) \]
are one-sided limits. The first means the limit as \( x \) approaches \( a \) from the left, and the second is the limit as \( x \) approaches \( a \) from the right. The limit
\[ \lim_{x \to a} f(x) \]
only exists if both one-sided limits exist and are equal.

**Problems:** Section 2.2 # 1, 5 and Section 2.4 # (1), 3, 5, (7), 19, 21