Limits

Study Guide

Problems listed in parentheses are for extra practice.

1. Limits by Factoring

Sometimes you can find a limit by factoring the numerator and/or denominator. For example:

$$\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}=\lim_{x\to 3}x+3=6.$$

Problems: Section 2.2 # 23, 25, (27), (29)

2. Conjugate Expressions

When taking the limit of an expression whose numerator or denominator includes a square root, it often helps to multiply through by the conjugate of the radical expression. For example:

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}=\lim_{x\to 9}\frac{x-9}{(x-9)\big(\sqrt{x}+3\big)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{6}$$

Problems: Section 2.2 # 37, 39, (41)

3. Limit Laws

You should understand the limit laws listed in Theorem 1 (pg. 66) and used in Example 5.

Problems: Section 2.2 # 51, 53, (55)

4. Sandwich Theorem

The sandwich theorem says that if f, g, and h are three functions and

$$f(x)\le g(x)\le h(x)$$

for all values of x close to a, and

$$\lim_{x\to a}f(x)=L\qquad\text{and}\qquad\lim_{x\to a}h(x)=L$$

for some number L, then we can conclude that

$$\lim_{x\to a}g(x)=L.$$

Problems: Section 2.2 # 63, (65a)

5. One-Sided Limits

The expressions

$$\lim_{x \to a^-} f(x) \qquad \text{and} \qquad \lim_{x \to a^+} f(x)$$

are one-sided limits. The first means the the limit as x approaches a from the left, and the second is the limit as x approaches a from the right. The limit

$$\lim_{x \to a} f(x)$$

only exists if both one-sided limits exist and are equal.

Problems: Section 2.2 # 1, 5 and Section 2.4 # (1), 3, 5, (7), 19, 21