Continuity and Limits Involving Infinity

Study Guide

Problems listed in parentheses are for extra practice.

1. Continuity

A continuous function is one whose graph can be drawn without lifting your pencil. The definition is that a function $f(x)$ is continuous at $x = a$ if:

(1) $f(a)$ is defined,

(2) $\lim\limits_{x\to a} f(x)$ exists, and

(3) $\lim\limits_{x\to a} f(x) = f(a)$.

Problems: Section 2.5 # 1, 3, (7), 13, (29), 31, 45

2. Limits as $x \to \infty$ or $x \to -\infty$

For most of these, it works pretty well to just imagine what happens when x is a very large number (or a very large negative number, if $x \to -\infty$). Remember that

$$\frac{\text{a reasonable number}}{\text{a very big number}} = \text{a very small number}$$

Sometimes it helps to divide through by the largest power of x that appears in the numerator or denominator. For example,

$$\lim_{x\to\infty} \frac{2x+5}{3x^2-7x+1} = \lim_{x\to\infty} \frac{\dfrac{2x}{x^2}+\dfrac{5}{x^2}}{\dfrac{3x^2}{x^2}-\dfrac{7x}{x^2}+\dfrac{1}{x^2}} = \lim_{x\to\infty} \frac{\dfrac{2}{x}+\dfrac{5}{x^2}}{3-\dfrac{7}{x}+\dfrac{1}{x^2}} = \frac{0+0}{3-0+0} = 0$$

Limits as $x \to \infty$ or $x \to -\infty$ correspond to horizontal asymptotes.

Problems: Section 2.6 # 3, 9, (11), 13, 15, (17), (19), 21

3. Vertical Asymptotes

A vertical asymptote happens when either

$$\lim_{x \to a^-} f(x) = \pm\infty \qquad \text{or} \qquad \lim_{x \to a^+} f(x) = \pm\infty.$$

These are common when the numerator of a fraction approaches something nonzero but the denominator approaches zero. Remember that

$$\frac{\text{a reasonable number}}{\text{a very small number}} = \text{ a very big number}$$

and you can tell whether the big number is positive or negative based on the signs of the numerator and denominator.

Problems: Section 2.6 # 37, 39, (41), (43), 49, (51), 53, (55)