More Derivatives

Study Guide

Problems in parentheses are for extra practice. If you haven't taken a calculus course before, you probably need a lot of practice taking derivatives, so I recommend doing quite a few of these extra problems.

1. Derivatives of Trigonometric Functions

You definitely need to know from memory that

$$\frac{d}{dx}\big[\sin x\big] = \cos x \qquad \text{and} \qquad \frac{d}{dx}\big[\cos x\big] = -\sin x.$$

There are also formulas for the derivatives of $\tan x$, $\cot x$, $\sec x$, and $\csc x$, but you probably don't need to memorize them.

Problems: Section 3.5 # (3, 5, 7, 9), 13, 45

2. The Chain Rule

The chain rule states that

$$\frac{d}{dx}\big[f(u)\big] = f'(u)\,\frac{du}{dx}$$

for any function f and any quantity u that depends on x. For example,

$$\frac{d}{dx}\big[\sin(x^3)\big] = \cos(x^3)\cdot 3x^2.$$

An equivalent formulation is

$$\frac{d}{dx}\Big[f\big(g(x)\big)\Big] = f'\big(g(x)\big)\,g'(x).$$

Remember that $\sin^k x$ means $(\sin x)^k$ as long as $k \neq -1$, and similarly for the other trig functions.

Problems: Section 3.6 # 23, 25, (27), 29, (31, 33), 35, (37, 39, 41, 51, 59), 67, 87, 89

3. Implicit Differentiation

You use implicit differentiation to find the slope of the tangent line to a curve defined by an equation involving x and y. The steps are:

(a) Take the derivative of the given equation with respect to x. This means you have to write a $\dfrac{dy}{dx}$ whenever you take the derivative of something involving y.

(b) Gather all the terms with $\dfrac{dy}{dx}$ on the left and all of the remaining terms on the right.

(c) Factor out a $\dfrac{dy}{dx}$ on the left and then solve for $\dfrac{dy}{dx}$.

Problems: Section 3.7 # 1, (3, 5, 7, 9), 11, (13), 15, (31), 43, 45

4. Derivatives of Inverse Functions

The inverse function theorem states that if a function f is differentiable at a point (a, b) then f^{-1} is differentiable at the corresponding point (b, a). Furthermore,

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'\big(f^{-1}(x)\big)}.$$

Problems: Section 3.8 # 7, 9

5. Derivatives of Logarithms

It follows from the inverse function theorem that

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}.$$

You can use this to differentiate any power using the identity

$$a^b = e^{b \ln a}$$

For example,

$$\frac{d}{dx}\left[x^x\right] = \frac{d}{dx}\left[e^{x \ln x}\right] = e^{x \ln x}\left(x\left(\frac{1}{x}\right) + (1)\ln x\right) = x^x(1 + \ln x).$$

Problems: Section 3.8 # (11, 13), 21, 25, 29, 67, 89, 93, (95, 99)