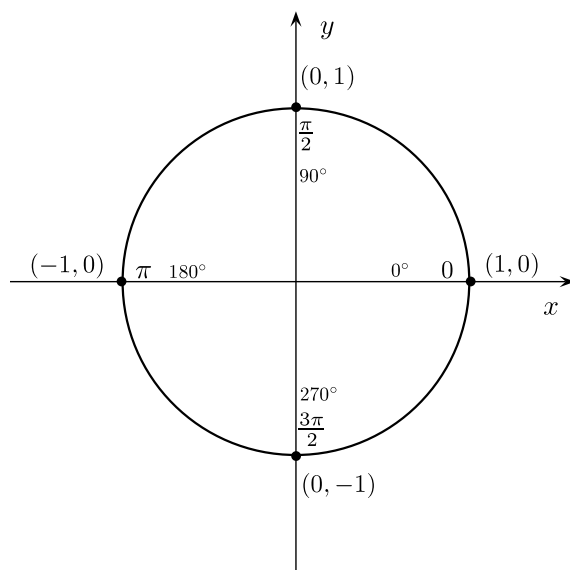


Trigonometry

Study Guide

1. Angles

We measure angles counterclockwise from the positive x -axis.



We almost always use radians instead of degrees in calculus. The conversion is:

$$\pi \text{ radians} = 180^\circ$$

In particular,

$$\frac{\pi}{6} = 30^\circ, \quad \frac{\pi}{4} = 45^\circ, \quad \frac{\pi}{3} = 60^\circ, \quad \frac{\pi}{2} = 90^\circ.$$

Negative angles are clockwise from 0. For example, $-\frac{\pi}{2}$ is the same as $\frac{3\pi}{2}$.

2. Trigonometric Functions

If (x, y) is a point on the unit circle at an angle of θ , then

$$\cos \theta = x \quad \text{and} \quad \sin \theta = y.$$

For example, the point $(0, 1)$ is at an angle of $\pi/2$, so

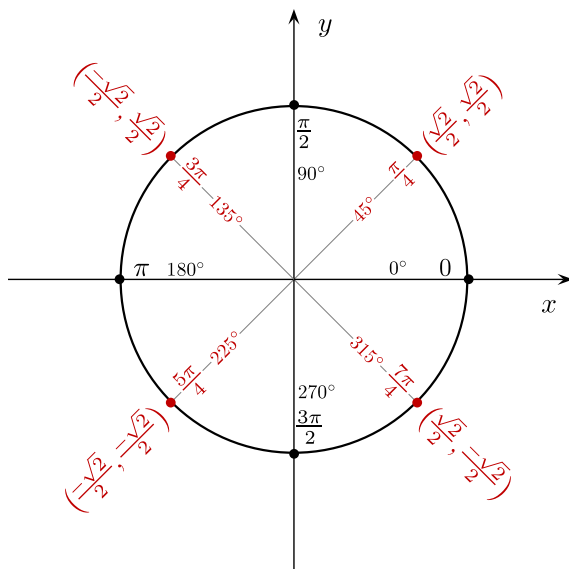
$$\cos\left(\frac{\pi}{2}\right) = 0 \quad \text{and} \quad \sin\left(\frac{\pi}{2}\right) = 1.$$

The other four trig functions are defined in terms of cosine and sine:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

3. Multiples of $\pi/4$

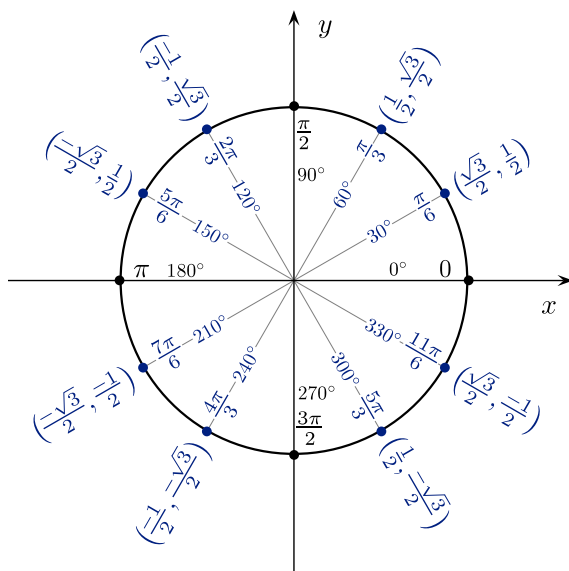
Points at certain multiples of $\frac{\pi}{4}$ have coordinates that $\pm \frac{\sqrt{2}}{2}$.



For example, $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$ and $\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

4. Multiples of $\pi/6$

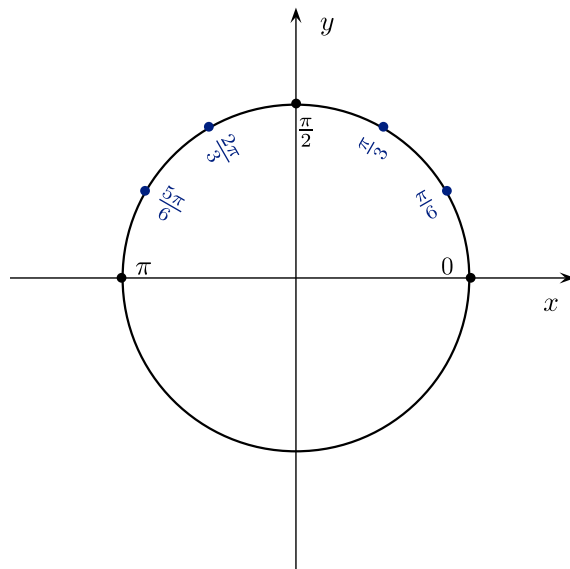
Points at certain multiples of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ have one coordinate which is $\pm \frac{1}{2}$ and one coordinate which is $\pm \frac{\sqrt{3}}{2}$.



Note that $\frac{\sqrt{3}}{2}$ is bigger than $\frac{1}{2}$, which helps you tell which coordinate is which.

5. Computing Values

Suppose you need to compute a value like $\tan\left(\frac{5\pi}{6}\right)$. The first step is to draw a unit circle and remember where $5\pi/6$ is:



Next we figure out the coordinates of the point labeled $5\pi/6$. It looks like the x -coordinate is negative and bigger than the y -coordinate, so

$$\cos\left(\frac{5\pi}{6}\right) = x = -\frac{\sqrt{3}}{2} \quad \text{and} \quad \sin\left(\frac{5\pi}{6}\right) = y = \frac{1}{2}.$$

Then

$$\tan\left(\frac{5\pi}{6}\right) = \frac{\sin(5\pi/6)}{\cos(5\pi/6)} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

6. Exercises

Use this technique to compute the following values. **Do not use a calculator**, and try drawing your own unit circles instead of referring to the ones above. The answers are on the next page.

1. $\cos(0)$
2. $\cos\left(\frac{\pi}{2}\right)$
3. $\sin\left(\frac{3\pi}{2}\right)$
4. $\sec(\pi)$
5. $\cot\left(-\frac{\pi}{2}\right)$
6. $\tan\left(\frac{\pi}{2}\right)$
7. $\cos\left(\frac{\pi}{4}\right)$
8. $\sin\left(\frac{\pi}{3}\right)$
9. $\tan\left(\frac{\pi}{3}\right)$
10. $\sec\left(\frac{\pi}{6}\right)$
11. $\csc\left(\frac{\pi}{4}\right)$
12. $\cot\left(\frac{\pi}{6}\right)$
13. $\cos\left(\frac{3\pi}{4}\right)$
14. $\sin\left(\frac{2\pi}{3}\right)$
15. $\sin\left(-\frac{\pi}{4}\right)$
16. $\sin\left(\frac{5\pi}{6}\right)$
17. $\sin\left(\frac{7\pi}{4}\right)$
18. $\cos\left(\frac{4\pi}{3}\right)$
19. $\tan\left(-\frac{\pi}{3}\right)$
20. $\csc\left(-\frac{2\pi}{3}\right)$
21. $\sec\left(\frac{7\pi}{4}\right)$
22. $\csc\left(\frac{11\pi}{6}\right)$
23. $\sec\left(\frac{2\pi}{3}\right)$
24. $\cot\left(-\frac{3\pi}{4}\right)$

7. Answers

1. 1

2. 0

3. -1

4. -1

5. 0

6. undefined

7. $\frac{\sqrt{2}}{2}$

8. $\frac{\sqrt{3}}{2}$

9. $\sqrt{3}$

10. $\frac{2\sqrt{3}}{3}$

11. $\sqrt{2}$

12. $\sqrt{3}$

13. $-\frac{\sqrt{2}}{2}$

14. $\frac{\sqrt{3}}{2}$

15. $-\frac{\sqrt{2}}{2}$

16. $\frac{1}{2}$

17. $-\frac{\sqrt{2}}{2}$

18. $-\frac{1}{2}$

19. $-\sqrt{3}$

20. $-\frac{2\sqrt{3}}{3}$

21. $\sqrt{2}$

22. -2

23. -2

24. 1