Math 4530 Homework 7. (short)

There are two ways to do great mathematics. The first is to be smarter than everyone else. The second is to be stupider than everyone else – but persistent

- R. Bott. (Bott made major contributions to algebraic topology, the subject we will next embark upon. He considered himself in the second category!)

Problems:

1. Show that gluing two Möbius strips along their boundary gives a topological space homeomorphic to a Klein bottle.

2. Let S^2 be the unit sphere $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ with the subset topology. Let $\sim$ be the equivalence relation on S^2 given by $w \sim v$ if $w = v$ or if $w = -v$. Let RP^2 denote the space $S^2/\sim$ with the quotient topology.

   Show that you can remove an open disc from RP^2 and get a space homeomorphic to a Möbius strip.

3. Recall that, if X and Y are surfaces, $X\#Y$ denotes the surface obtained by removing an open disc from X and and open disc from Y and gluing them along their boundaries.

   Prove that $(\mathrm{RP}^2\#\mathrm{RP}^2)\#\mathrm{RP}^2$ is homeomorphic to $(S^1 \times S^1)\#\mathrm{RP}^2$.

4. This question is an exercise that we will need to use next week.
   Let $T \subset \mathbb{R}^2$ be the union of $[-1, 1] \times \{0\}$ and $\{0\} \times [-2, 0]$, with the subset topology from $\mathbb{R}^2$. (it looks like a letter T).

   (a) Write a formula that defines a continuous function $f : [0, 1] \times T \to T$ so that $f(0, t) = t$ for every $t \in T$, and so that $f(1, t)$ is the point on the y-axis with the same y coordinate as t. (so if $t = (x, y)$ as a point in $\mathbb{R}^2$, then $f(1, t) = y$).

   You do not need to *prove* the function is continuous provided that a calculus student could easily just check it.

   (b) Draw a picture of the set $\{f(1/2, t) : t \in T\}$.