# Math 4530 Homework 9: covering spaces

*Problems worthy of attack prove their worth by fighting back*
- Piet Hein

**Problems:**

For the first two questions, please think of $S^1$ as $\mathbb{R}/\sim$ where $x \sim y$ if $x = y + k$ for some $k \in \mathbb{Z}$. Thus, the function $\Delta$ defined on the previous homework takes integer values.

1. Suppose $f : S^1 \to S^1$ is a continuous map. Show that there is a map $g$ that is homotopic to $f$, and with $g(0) = 0$. Hint: compose with rotations...

2. (a) Suppose that $f : S^1 \to S^1$ satisfies $\Delta(f) = \Delta(id)$, where $id$ is the identity function $x \mapsto x$. Show that $f$ is homotopic to $id$. Hint: the previous question might be helpful!

   (b) Suppose that $g : S^1 \to S^1$ satisfies $\Delta(g) = 0$. Show that $g$ is homotopic to a constant map.

3. Define a covering map $S^2 \to \mathbb{R}P^2$ and prove that it is a covering map. What is the degree of the cover?

   CHALLENGE/BONUS: (not required to hand in)
   Does there exist a degree two covering map $S^2 \to S^2$? What about degree 3?

4. Let $Y$ be a topological space that is homeomorphic to the numeral 8. You may take $Y$ to be the set $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1 \text{ or } x^2 + (y - 2)^2 = 1\}$.

   Does there exist a covering map from $Y$ to $S^1$? Either define the map and show it is a cover, or prove that none exists.

5. Suppose that $\pi : \mathbb{R} \to \mathbb{R}$ is a covering map. Show that $\pi$ is necessarily a homeomorphism.