Math 130 Homework 10 – final homework set.

Upcoming deadlines:

- This homework is due Nov. 24
- Thursday, Nov. 19: Practice presentations + written draft of independent project due in class.
- Nov 24, Dec 1, Dec 3: In-class presentations.
- Dec. 3: Written independent project due.

Reading:

- Stillwell, chapter 8.
- Many images of tilings of hyperbolic space by Jos Leys:
  `http://www.josleys.com/show_gallery.php?galid=325`
- (related to 8.7 in S4P) Some movies of isometries of hyperbolic space by Goodman-Strauss
  `http://comp.uark.edu/~strauss/hyperbolia/gallery2.html`

1. Do the following problems from S4P: 8.4.2, 8.4.3
2. *Area of spherical triangles.* Following our discussion from class on Thursday, do the following problems from S4P: 8.5.1-8.5.5
3. *Area of spherical and hyperbolic regions*
   (a) Derive a formula for the area of a 4-sided region on the unit-radius sphere. Your formula should give the area in terms of the angles at the vertices. (hint: divide the region up into triangles).
   (b) Generalize your formula above for a n-sided region on the sphere.
   (c) Now using the fact that the area of a hyperbolic triangle is $\pi - (A + B + C)$, where A, B, C are the angles, prove the following formula for hyperbolic polygons:
   If P is a polygon in $\mathbb{H}^2$, then
   $$\text{Area}(P) = -2\pi + (\text{sum of external angles})$$
   How does this compare with your formula for the sphere?
4. Do the following problems from S4P: 8.6.1, 8.6.2, 8.6.4–8.6.6
5. Do the following problems from S4P: 8.7.1, 8.7.2.
6. *This problem will be introduced in class on Tuesday, Nov. 17.* Build a paper model of hyperbolic space with 7 triangles around a vertex. A template for equilateral triangles is posted on the website.
   Now draw some geodesics (straight lines) on the model. To draw a geodesic starting in a given direction, draw a straight line inside a triangle (don't aim for the corners!) and when you get to the edge, flatten the edge momentarily and continue the straight line on the other side.
   (a) Draw two geodesics that start off as parallel segments, but eventually diverge.
   (b) Can you draw a (big) geodesic sided rectangle? (i.e. four sides, 90 degree corners). Drawing it inside a single flat triangle piece is cheating. If not, why not?
   (c) Draw the image of some of your geodesic segments on the Poincaré disc by copying them off of your paper model onto the image on the following page. Can you describe what they look like?
   (d) Estimate the area of a disc of radius r on the hyperbolic plane by counting triangles within a disc of fixed radius, and looking for a pattern. This is a very open-ended question, but I expect that you should be able to make an argument that a disc of radius r (for r large) has area at least as big as $\lambda 2^r$, for some constant λ. Perhaps you can even do better.

   You do not have to hand in your paper model, just your answers to parts b-d