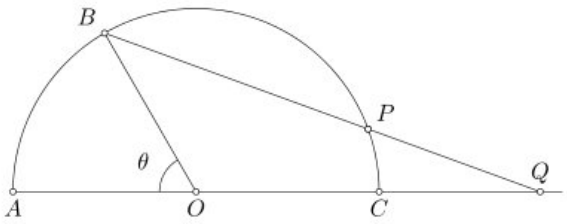
Math 130 Homework 2

Reading:

- S4P Chapter 2, including "discussion" at the end of 2.9.
- 111 ways to prove the pythagorean theorem: `http://www.cut-the-knot.org/pythagoras/index.shtml` (no need to read all of them!)
- Short critique of the method of superposition (discussed in class on Tuesday, posted on website)

1. (Archimedes trisection of the angle using *ruler* and compass.)
   Prove that you can trisect an angle using ruler and compass, using the following strategy. (I expect you to write a complete proof).
   Given angle θ and a unit length marking on your ruler, first construct a unit radius semi-circle. Then slide your ruler until you find a segment of length 1 (PQ in the picture) between the base and circumference of the semicircle. Show that $\measuredangle PQC$ is one third of θ.



2. Look at some of the proofs of the Pythagorean theorem at
   `http://www.cut-the-knot.org/pythagoras/index.shtml`
   (there are 111 proof ideas there, scroll down to find them). Pick one proof that looks *interesting* to you, and give a clear, step-by-step exposition of the proof. The website doesn't give full proofs of everything – and when it does, the explanation is not always very good. Choose something that you think you can explain better than the website does, and write your best proof.
   (You may *not* do the decomposition proof that we did in class, but anything else is okay).

3. Do the following problems from S4P:
   2.3.2, 2.3.3 (you do not need to give a proof, just an illustration and an explanation why it works)
   2.5.2, 2.5.5 (you may find it helpful to try 2.5.4 first)
   2.7.5, 2.8.1-3.

4. In class, we showed that the angle sum of a triangle is π or $180°$.
   (a) Use this to find the angle sum of a convex n-gon. Is your formula true for non-convex polygons?
   (b) Using b), prove that the only convex regular (all angles and sides equal) polygons that tile the plane are the triangle, square and hexagon.
   (c) (***Challenge problem: not to hand in!) Show that no convex 7-gon (not necessarily regular) can tile the plane.
   remark: understanding the convex pentagons that tile the plane is an interesting problem! Perhaps you've heard about it in the news lately

Not homework, but for your reference: S4P Problems 2.7.1-4 give an outline of how to prove the Bolyai–Gerwien decomposition theorem that we'll do in class on Thursday. You may find it helpful to do this as a review. This will give a slightly different proof.