Math 130 Homework 3

Reading:

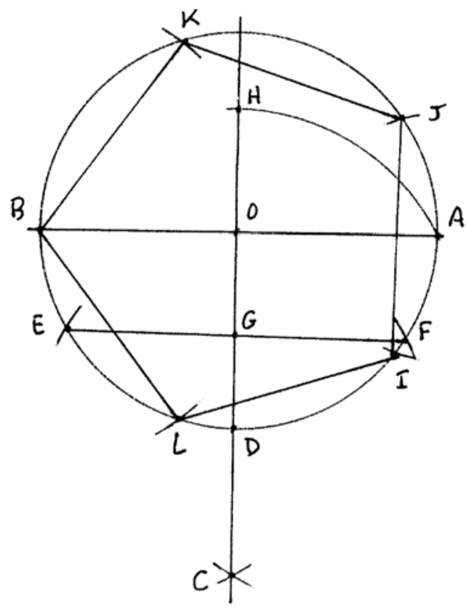
- Euclid's construction of the regular pentagon. (On website. Compare with your HW from last week)
- Constructible regular polygons and field extensions reading (link on website)
- Optional videos:
  1) Prof. David Eisenbud (who should obviously be teaching this class!) constructs a regular 17-gon. `http://www.youtube.com/watch?v=87uo2TPrsl8`. The construction starts at about 3:45.
  2) Prof. Eisenbud again, this time actually explaining Gauss: `http://www.youtube.com/watch?v=oYlB5lUGlbw`

1. Prove that the construction below gives a regular pentagon.

   1. Draw any line through O. Get A, B.
   2. Circle AB.
   3. Circle BA, get C.
   4. OC, get D.
   5. Circle DO. Get E, F.
   6. EF, get G.
   7. Circle GA, get H.
   8. Circle center A, radius OH, get I, J.
   9. Circle center B, radius IJ, get K, L.

   10–14. Draw BK, KJ, JI, IL, LB.

   Then $BKJIL$ is the required pentagon.



2. Prove that the triangle formed by the sides of an inscribed hexagon, pentagon, and decagon is right angled. (see picture on back for an illustration – not a suggested method of proof!) You may use any high-school level geometry in your proof.

3. Using the previous problem, show that the segment AH in problem 1 has the same length as the side of the pentagon.

4. **On algebraic numbers:** Recall that a number is *algebraic* if it is the root of a polynomial in $\mathbb{Q}[x]$. It is *algebraic of degree* n if the lowest degree polynomial of which it is a root has degree n.
   (a) Prove that if a is algebraic of degree n, then $\sqrt{a}$ is algebraic of degree at most $2n$. Must it be exactly $2n$?
   (b) Prove that if a is algebraic of degree n, then a^2 is algebraic of degree at most n. Must it be exactly n?
   (c) Prove that $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2})(\sqrt{3})$, and conclude that this field has degree 4 over $\mathbb{Q}$.

5. Complete our work from class: show that
   (a) If L_1 and L_2 are line segments whose endpoints have coordinates in a field F, then their intersection has coordinates in F.

(b) if $(x-a)^2 + (y-b)^2 = r^2$ and $(x-c)^2 + (y-d)^2 = s^2$ are two circles, with $a, b, c, d, r, s \in F$, then a point (x, y) where the circles intersect is the root of a degree 2 polynomial with coefficients in F. [possible hint: replace one equation with the difference of the two equations to reduce the problem to the intersection of a circle and straight line].

6. Prove that the regular 9-gon is not constructible. You may use results proved in class.

7. More generally, prove that an angle of n degrees, where $0 < n < 180$ is an *integer* is constructible if and only if n is a multiple of 3.

8. Show, without using Gauss' theorem, that if you can construct a regular k-gon and a regular n-gon, where k and n are relatively prime, then you can construct a regular kn-gon

Picture for Question 2: