Math 130 Homework 7

Reading:

- Stillwell, 5.8, beginning of chapter 6 (we will only cover some parts of 6.1-6.3, then skip to ch.7).

Don't forget: your project proposal is due on Thursday, Oct. 27

1. Do the following problem from S4P: 5.7.3
   (note, e here is just some number, not the base of the natural logarithm)

2. (optional, not to hand in) Do 5.8.4-5.8.6. in S4P. Then deduce from 5.8.6 that there are four ways of permuting p, q, r, s that leave the cross ratio invariant. Use this to justify your work in the last question of the previous problem set. Does this make that question easier?

3. Building on your work from the previous problem set, show that the map that associates an invertible matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ to the function $x \mapsto \frac{ax+b}{cx+d}$ is a homomorphism from $GL_2(\mathbb{R})$ to the group of linear fractional functions. What is the kernel of this homomorphism?

4. There is a similar homomorphism from $GL_2(\mathbb{C})$ to the group of complex linear fractional transformations. What is its kernel?

5. Let $\mathbb{F}_3$ be the field with three elements. $\mathbb{F}_3 = \{0, 1, 2\}$ with addition and multiplication mod 3. Following the discussion in section 5.9 of S4P, calculate how many elements there are in the projective plane $\mathbb{F}_3\, P^2$

6. **Duality.** In HW5 (which you now have returned to you), you proved that in projective geometry, any point has 3 lines passing through it. Use that fact to prove the following:

   Suppose P is a set of points, and L is a collection of subsets called lines, satisfying the axioms of projective geometry. Define a set of new points $P!$ (call them "points!") to be the elements of L, and say that two points! lie on a line! if they intersect (so a line! consists of all the points! passing through a common point). Show that the points! and lines! satisfy the axioms P1-P4 of projective geometry.

7. A consequence of 6 is that every theorem in projective geometry has a *dual statement* where you interchange the role of points and lines. What is the dual statement of the Projective Pappus Theorem (in 6.1)?