Math 130 Just For Fun challenge problem 1

On equidecomposability

Not-to-hand-in (unless you solve problem b and want to tell me about it):
Say that a polygon is *square-decomposable* if it can be partitioned into finitely many non-overlapping squares (the squares are allowed to be of different sizes). For example, this picture below shows a square-decomposition of a rectangle.

(1) (not hard) Suppose a rectangle R has side lengths a and b, where the ratio a/b is a rational number. Show that R is square-decomposable.

(2) (serious challenge) Prove the converse to the above statement. Namely, if a rectangle of side lengths a and b is square-decomposable, then a/b is rational.