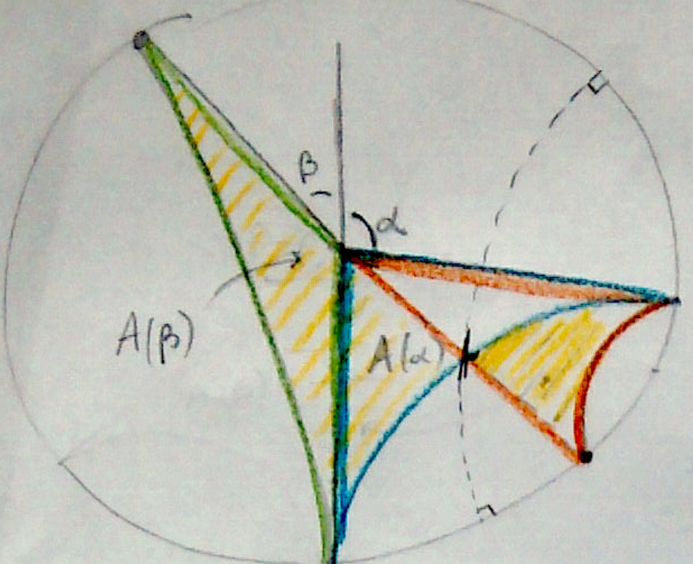
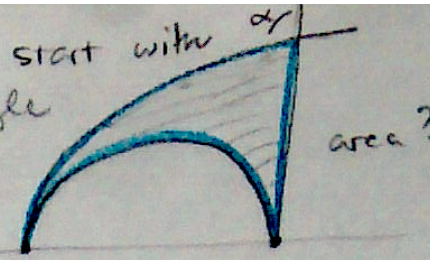
Area of hyperbolic triangles



1. start with one-angle region



Let $A(\alpha)$ = area of one-angle region with exterior angle α

2. show $A(\alpha+\beta) = A(\alpha) + A(\beta)$ for any α, β

<u>Proof</u>: reflect in / to show yellow regions are <u>isometric</u> (same area)

since $A(\alpha) + A(\beta) = A(\alpha+\beta)$

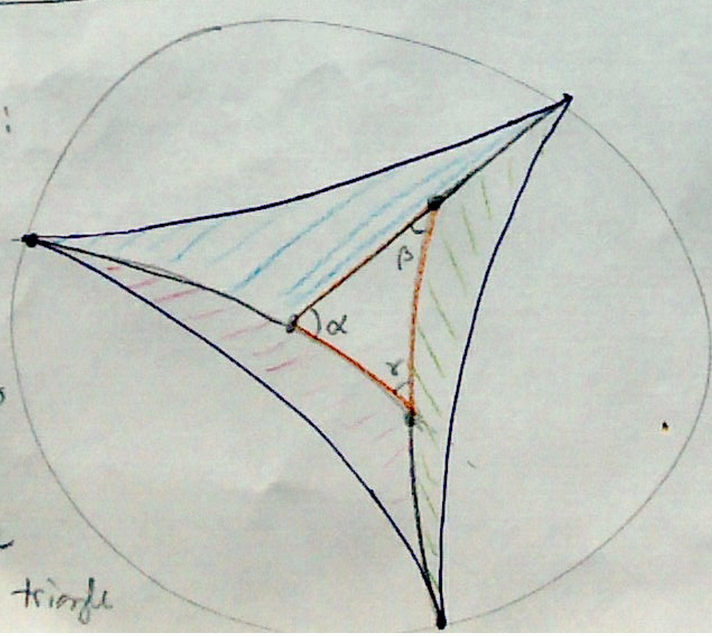
and $A(\alpha)$ is <u>Continuous</u>, have $A(\alpha) = c \cdot \alpha$ (<u>linear</u>)

When $\alpha = \pi$, get 0-angle triangle (vertices on boundary) so $A(\pi) = \pi$ and $c = 1$.

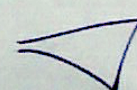
(in general '<u>curvature</u>' of space changes c), like radius. compare with area of spherical triangles.

Conclude $A(\alpha) = \alpha$.

3. General triangle: with straight line sides.



Extend sides to edge of disc and place inside 0° angle triangle

 = area π

$\Delta = \pi - \alpha - \beta - \gamma$ by one-angle region formula.