

# Topics/Reading for students

A giant list, work in progress...

## Background: Geometric group theory / hyperbolic groups

Books for reference:

- De La Harpe's *topics in geometric group theory*. Classic reference.
- Ghys and De la Harpe *Sur les Groupes Hyperboliques d'après Mikhael Gromov* (excellent book, but in French)
- Drutu and Kapovich have a more recent very long and detailed book.

Surveys/courses

- Neumann and Shapiro, *a short course geometric group theory* (available on Neumann's website)
- Gromov's *Asymptotic invariants of infinite groups* <https://www-cambridge-org.proxy.library.cornell.edu/core/books/geometric-group-theory/7BB8B91659BF3840E36429061A3983A5> (sometimes vague but highly inspirational)
- Gromov's Hyperbolic groups paper. (same)
- Ghys's Bourbaki survey on hyperbolic groups
- Ghys's Bourbaki survey on random groups *groupes aleatoires*. I particularly like this one! [http://archive.numdam.org/article/SB\\_2002-2003\\_\\_45\\_\\_173\\_0.pdf](http://archive.numdam.org/article/SB_2002-2003__45__173_0.pdf)
- Kapovich *Lectures on Quasi-Isometric rigidity* [https://www.math.ucdavis.edu/~kapovich/EPR/pc\\_lectures3.pdf](https://www.math.ucdavis.edu/~kapovich/EPR/pc_lectures3.pdf) Gives Tukia's proof of Mostow using the quasi-conformal structure at the boundary. Has a version in the Drutu-Kapovich book also, some of Tukia's papers also readable.
- Quint: An introduction to random walks on groups <https://www.math.u-bordeaux.fr/~jquint/publications/CoursChili.pdf>. Lots of good material here, especially the definition of *amenability*.

## Background: 3-Manifolds / geometry

Basics:

- Scott, Geometries of 3-manifolds (classification of the eight 3-dimensional geometries)
- Thurston's book: additional reference for the above. Also: hyperbolic geometry, Margulis Lemma / thick-thin decomposition.
- Hatcher's basic 3-manifold topology (prime decomposition, JSJ decomposition).
- Neumann's *Notes on geometry and 3-manifolds* <https://www.math.columbia.edu/~neumann/preprints/budfinal.pdf>. I prefer this to Hatcher.

Topics from Thurston's notes: <http://library.msri.org/books/gt3m/>

- Hyperbolic Dehn filling
- Flexibility and rigidity of geometric structures, Kleinian groups and quasi-fuchsian groups, deformations of Kleinian groups
- Mostow rigidity, see below.

Other

- Thurston's *A norm on the homology of 3-manifolds* paper.
- Follow up to above: The *universal circle action* for a quasigeodesic flow, pseudo-Anosov flow, or taut essential lamination. See Steven Frankel's papers for QG flows, also Calegari-Dunfield. Related: Thurston's *3 manifolds that slither over the circle*

## Groups acting on the circle and the plane

- Survey: Ghy's *groups acting on the circle*
- Convergence groups are Fuchsian groups. Paper of Gabai. Bowditch's papers for the definition of convergence group.
- Calegari's *Circular groups, planar groups and the Euler class*
- My papers with Maxime Wolff (start with the one in PJM, then the mapping class group paper).

- Related to 3-manifolds topic: The *universal circle action* for a quasigeodesic flow, pseudo-Anosov flow, or taut essential lamination. See Steven Frankel's papers for QG flows, also Calegari–Dunfield and Thurston's preprint *Three-manifolds, Foliations and Circles, I*.
- Navas' book: Groups of circle diffeomorphisms

## Symmetric spaces, nonpositive curvature and rigidity

Mostow rigidity:

- **Mostow/Gromov** Thurston notes, Benedetti–Petronio, Martelli all have versions of this
- Munkholm's survey paper on Gromov's version <https://link.springer.com/chapter/10.1007/BFb0099242>
- **Barycenter method** Besson–Courtois–Gallot and the barycenter method *Minimal entropy and Mostow's rigidity theorem*
- **Quasiconformal method** Tukia's proof, given in Drutu and Kapovich's geometric group theory book, chapter 22; Gehrig and Martin's book "An Introduction to the Theory of Higher-Dimensional Quasiconformal Mappings" has a nice self-contained chapter giving a proof of Mostow using this argument.
- **Too low dimension** "Mostow rigidity on the line" is the title of a survey paper of S. Agard, outlining results of Tukia, Sullivan, and others.
- **Local rigidity** Beregeron–Gelander's note on local rigidity.

Books for broader reference on semi-simple groups:

- Witte Morris' book on arithmetic groups.
- The appendix to Ballman–Gromov–Schroeder gives an intro on Weyl Chambers and does an example for  $SL(n, \mathbb{R})$ . This is good to know!

Other rigidity theorems:

- Ghys' actions of lattices on the circle (french). Any action of a lattice in a semi-simple Lie group basically factors through a surjection of that group onto  $PSL(2, \mathbb{R})$ . <http://perso.ens-lyon.fr/ghys/articles/actionsreseaux.pdf>. There is an outline of the proof in Ghys' *groups acting on the circle* survey paper, it is a nice way to get into the theory of lattices in semi-simple Lie groups.
- Rich Schwartz's *Pattern rigidity*
- Further reading on the Barycenter method related to BCG: Connel, Farb *Some recent applications of the barycenter method in geometry* <https://arxiv.org/abs/math/0204093>

## Dynamics in nonpositive curvature (some overlap with previous section!)

Books for reference:

- Ballman's *Lectures on spaces of nonpositive curvature*. Contains an appendix with proof of ergodicity of geodesic flow.
- Bridson–Haefliger is a standard reference text on NPC spaces, as is Ballman–Gromov–Schoeder. Key topics: Buseman functions, Tits metric on boundary.

Theorems:

- Otal's marked length spectrum rigidity. Survey paper/ minicourse notes of Wilkinson: [http://www.math.utah.edu/pcmi12/lecture\\_notes/wilkinson.pdf](http://www.math.utah.edu/pcmi12/lecture_notes/wilkinson.pdf). Nice notes.
- Ergodicity of geodesic flow from BGS

## Anosov flows on 3-manifolds

- Lecture notes of Barthelme: <https://drive.google.com/file/d/1Jmfx1As6i6f8YXAERFWHz-HVhd0mrvPv/view>
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Other possible directions... references TBA

Mapping class groups and Teichmüller space, mapping class groups of infinite type surfaces, left-orderable groups, rigidity theorems for “big” groups of homeomorphisms...