Math 1272 Quiz 10 May 1, 2014 Name SOLUTIONS

Time allow: 20 minutes

1.(35%) Given three vectors $\mathbf{a} =< 1, 1, 0 >$, $\mathbf{b} =< 4, 2, 2 >$, and $\mathbf{c} =< 4, 3, 3 >$.

(i) Find the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$.

(ii) Find the volume of the parallelpiped determined by the vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$.

(i) area is $|\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{vmatrix} = \langle 2, -2, -2 \rangle$$

$$|\vec{a} \times \vec{b}| = \sqrt{2^2 + (-2)^2 + (-2)^2} = \sqrt{12} = 2\sqrt{3}$$

(ii) volume is $|\vec{c} \cdot (\vec{a} \times \vec{b})|$

$$|\langle 4, 3, 3 \rangle \cdot \langle 2, -2, -2 \rangle| = |8 - 6 - 6|$$

$$= |-4|$$

$$= 4$$

2. (15%) Find the angle between the vectors $\mathbf{a} =< 4, 0, 2 >$ and $\mathbf{b} =< 2, -1, 0 >$.

$$\vec{a} \cdot \vec{b} = |\vec{a}| \, |\vec{b}| \cos\theta \Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \, |\vec{b}|}$$

$$|\vec{a}| = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$|\vec{b}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$\vec{a} \cdot \vec{b} = 8 + 0 + 0$$

$$\cos\theta = \frac{8}{\sqrt{20}\sqrt{5}} = \frac{8}{\sqrt{100}} = \frac{8}{10} = \frac{4}{5}$$

PLEASE SEE THE OTHER SIDE FOR MORE PROBLEMS

3. (20%) Find the plane through the point $(5, 3, 5)$ and perpendicular to the vector $< 2, 1, -1 >$.

$$2(x-5) + (y-3) - (x-5) = 0$$

or

$$2x + y - z = 8$$

4. (30%) True or False. Fill in True or False.
Let **a**, **b**, and **c** be three-dimensional non-zero vectors.

(i) F $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$ is a scalar.

(ii) T $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ is a scalar.

(iii) T If $\mathbf{a} \times \mathbf{b} = 0$, then $\mathbf{a}$ and $\mathbf{b}$ are parallel.

(iv) T The two lines: $L_1 : x = 1 + t, y = 1 - t, z = 2t$, $L_2 : x = 2t, y = -2t, z = 7 + 4t$, are parallel.

(v) F If a plane is parallel to the line $x = 1 + t, y = 1 - t, z = 2t$, then the normal vector of the plane is parallel to the vector $< 1, -1, 2 >$.

(vi) T If a plane P_1 is parallel to the plane $P_2 : x - y + 2z = 4$, then the normal vector of P_1 is parallel to the vector $< 1, -1, 2 >$.

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