MATH 1271 Spring 2014 | Discussion Quiz 06 | Name: SOLUTIONS
March 27, 2014 | 20 minutes | Lecture 030 Section _____

1. Which of the following sequences or series are convergent? Circle all correct answers. No justification is required. 30%

a) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

b) $\sum_{n=1}^{\infty} \frac{3 \cdot 5^n}{4^n}$

c) $\{a_n\}$ where $a_n = \frac{2n-1}{3n+1}$

d) $\sum_{n=1}^{\infty} a_n$ where $a_n = \frac{2n-1}{3n+1}$

e) $\sum_{n=1}^{\infty} 3\left(\frac{4}{5}\right)^n$

f) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

2. Determine if the following series is convergent or divergent. If it is convergent, determine the sum. (*HINT:* What is the sum of the first 4 terms? What is the sum of the first n terms?) 25%

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$$

$$S_n = 1 + \frac{1}{2} - \frac{1}{(n-1)+2} - \frac{1}{n+2}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right) = \lim_{n\to\infty} S_n = \frac{3}{2}$$

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3. Determine if the following series are convergent or divergent by using the INTEGRAL TEST.

$$\sum_{n=5}^{\infty} \frac{1}{n \cdot \ln(n) \cdot \ln(\ln(n))}$$

25%

u-sub. twice

$$\int_5^{\infty} \frac{dx}{x \cdot \ln x \cdot \ln(\ln(x))} \overset{\downarrow}{=} \ln\left(\ln\left(\ln(x)\right)\right)\Big|_5^{\infty}$$

$$= \infty$$

So the series is **divergent**

$$\sum_{n=1}^{\infty} \frac{1}{9n^2+4}$$

20%

$$\int_1^{\infty} \frac{dx}{9x^2+4}$$

$9x^2 = 4u^2 \Rightarrow x = \frac{2}{3}u$

$dx = \frac{2}{3}du$

$$= \int_{3/2}^{\infty} \frac{\left(\frac{2}{3}du\right)}{4u^2+4} = \frac{1}{6}\arctan u\Big|_{3/2}^{\infty}$$

$$= \frac{1}{6}\left[\lim_{u\to\infty}\arctan(u) - \arctan 3/2\right]$$

$\lim_{u\to\infty}\arctan(u) = \pi/2$

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$< \infty$

So the series **converges**