Math 1272 Lecture 30 Quiz 7

February 6, 2014
Name: SOLUTIONS

Time allowed: 20 minutes. Good luck!

1. (*30* %) Is the series

$$\sum_{n=3}^{\infty} \frac{(\sin n)^2}{n(n+3)}$$

convergent or divergent? Show all work and/or explain your reasoning fully. State the names of any theorems you use.

Converges: Comparison Test

$$\frac{(\sin n)^2}{n(n+3)} \leq \frac{1^2}{n(n+3)} \leq \frac{1}{n^2}$$

$\sum_{n=3}^{\infty} \frac{1}{n^2}$ converges by the Integral test (p-test).

2. (*20%*) Consider the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}.$$

Which of the following statements is **incorrect**?

a) It converges when $p > 0$. TRUE

b) It diverges when $p < 1$. FALSE

c) It is absolutely convergent when $p > 1$. TRUE

d) It is conditionally convergent when $0 < p < 1$. TRUE

e) It diverges when $p = 0$. TRUE

3. (*35 %*) For the follwoing series, state whether or not you can use the Alternating Series Test to determine convergence. If not, explain why not.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+5}$. No. $b_n = \frac{n^2}{n^2+5}$ $\lim_{n\to\infty} b_n = 1$

b) $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$. Yes, you can use the AST

c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{100}}$ Yes, you can use the AST

4. (*15 %*) Indicate which of the following series are **divergent**

a) $\sum_{n=1}^{\infty} \frac{n!}{5^n}$. Divergent. $\lim_{n\to\infty} \frac{n!}{5^n} = \infty \neq 0$

b) $\sum_{n=1}^{\infty} \frac{9^n}{n(-2)^{n+1}}$. Divergent. Ratio test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{9^{n+1}}{(n+1)2^{n+2}} \cdot \frac{n2^{n+1}}{9^n}\right|$$

$$= \lim_{n\to\infty} \frac{9}{2}\left(\frac{n}{n+1}\right) = \frac{9}{2} > 1$$

c) $\sum_{n=3}^{\infty} \frac{(-12)^n}{n}$.

Divergent.

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{12^{n+1}}{n+1} \cdot \frac{n}{12^n}$$

$$= 12 > 1$$