Math 1272 Lecture 30 Quiz 8

April 10, 2014
Name: SOLUTIONS

1. (30%) For each of the following, Decide whether the series (A) converges absolutely, (B) converges conditionally, (C) equals ∞, (D) equals $-\infty$, or (E) is undefined. No justification is required.

B   I. $\sum_{n=0}^{\infty} (-1)^n / \sqrt[7]{n+1}$.

A   II. $\sum_{n=2}^{\infty} \frac{1}{n \log^3 n}$.   Integral Test

$$\int_2^{\infty} \frac{dx}{x \log^3 x} = \int_{\log 2}^{\infty} \frac{du}{u^3}$$ which converges by the p-test since $3 > 1$.

$u = \log x$
$du = \frac{1}{x} dx$

A   III. $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$.   Ratio Test

$$\lim_{n\to\infty} \left| \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n n!} \right| = \lim_{n\to\infty} \left| 2 \frac{\cancel{(n+1)!}}{\cancel{n!}} \frac{n^n}{(n+1)^{\cancel{n+1}}} \right|$$

$$= 2 \underbrace{\lim_{n\to\infty} \frac{n^n}{(n+1)^n}}_{1/e} = \frac{2}{e} < 1$$

2. (15%) Which of the following is **not** possible for the power series $\sum_{n=0}^{\infty} b_n (x-a)^n$? No justification is required.

(A) It converges for all x.

(B) For some $R > 0$, it converges for $|x-a| < R$ and diverges for $|x-a| \geq R$.

(C) For some $R \geq 0$, it converges for $|x-a| \leq R$ and diverges for $|x-a| > R$.

(D) For some $R > 0$, it converges for $-R \leq x-a < R$ and diverges for all other x.

(E) It converges for no x.

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3. Consider the function

$$f(x) = \frac{4}{1-9x^2}.$$

(a) (35%) Find a power series representation for $f(x)$.

$$4\sum_{n=0}^{\infty} 9^n (x^2)^n$$

(b) (20%) Where does the series from part (a) converge?

* Radius of convergence:

$$\lim_{n\to\infty} \left|\frac{9^{n+1}\, x^{2n+2}}{9^n\, x^{2n}}\right| = \lim_{n\to\infty} 9|x^2| < 1$$

$$\Rightarrow x^2 < \frac{1}{9} \Rightarrow |x| < \frac{1}{3}$$

* Check endpoints:

$x=\frac{1}{3}$: $4\sum_{n=0}^{\infty} 9^n \left(\left(\frac{1}{3}\right)^2\right)^n = 4\sum_{n=0}^{\infty} 9^n\left(\frac{1}{9}\right)^n = 4\sum_{n=0}^{\infty} 1 = \infty$

$x=\frac{-1}{3}$: $4\sum_{n=0}^{\infty} 9^n \left(\left(\frac{-1}{3}\right)^2\right)^n = 4\sum_{n=0}^{\infty} 9^n\left(\frac{1}{9}\right)^n = \infty$

$\boxed{\text{Converges on } \left(\frac{-1}{3}, \frac{1}{3}\right)}$

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