MATH 1272
Midterm II
Professor Bramson
October 27, 2011

Name:_____

T.A.:_____

INSTRUCTIONS: There are 100 points possible on the exam. Do as many problems as you can. Answers must be in the boxes provided to be counted. Show your work in the space provided below each problem - outside the boxes. If you need extra space, state where the work is being done. Also, be sure to justify your answers. Note that some formulas are given on page 7. Page 6 is provided as scratch; be sure your exam has all 7 pages. **NO CALCULATORS** are permitted. GOOD LUCK!

1. (15 pts) What are **all** of the polar coordinates that correspond to

$$(x, y) = (2\sqrt{3}, 2)?$$

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2. (15 pts) Compute $\int \frac{1}{1-\sin 2\theta} d\theta$. Show your work.

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3. (15 pts) Find the surface area of the solid of revolution of the curve given by $x = 2t^3$, $y = t^3 + \sin t$, from $t = 0$ to $t = 1$, by rotating the curve around the line $x = -2$. Write your answer as an integral, which you should not bother to compute. (Simplify as much as possible within the integral.) Show your work.

4. (15 pts) Find the general solution of the differential equation $y' = (x+y)^2 - (x-y)^2$. Write the answer as a function of y. Show your work.

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5. (10 pts) Applying Euler's formula to the equation $y' = g(x)$, what formula should you use for y_6, given that $x_0 = 3$, $y_0 = 4$, increments are of size .5, $g(5.5) = .3$, $g(6) = .2$, and $g(6.5) = .1$? (No justification is required.)

(A) $y_6 = y_5 + .05$

(B) $y_6 = y_5 + .1$

(C) $y_6 = y_5 + .15$

(D) $y_6 = y_5 + .2$

(E) $y_6 = y_5 + .3$

(F) $y_6 = 4y_5 + .2$

6. (10 pts) Suppose that a tank initially contains 50 lbs. of salt dissolved in 3,000 gallons of water. Brine that contains 1 lb. of salt for each 4 gallons of water enters the tank at a rate of 80 gallons per minute. The solution is kept thoroughly mixed and drains from the tank at 25 gallons per minute. If $y(t)$ denotes the amount of salt in pounds in the tank at time t, which differential equation does $y(t)$ satisfy? (No justification is required.)

(A) $y' = 20 - \frac{25y}{3{,}000-55t}$

(B) $y' = 20 - \frac{25y}{3{,}000+55t}$

(C) $y' = 20 - \frac{25y}{3{,}000}$

(D) $y' = 20 + \frac{25y}{3{,}000}$

(E) $y' = 80 - \frac{25y}{3{,}000-55t}$

(F) $y' = 80 - \frac{25y}{3{,}000+55t}$

(G) $y' = 80 - \frac{25y}{3{,}000}$

(H) $y' = 80 + \frac{25y}{3{,}000}$

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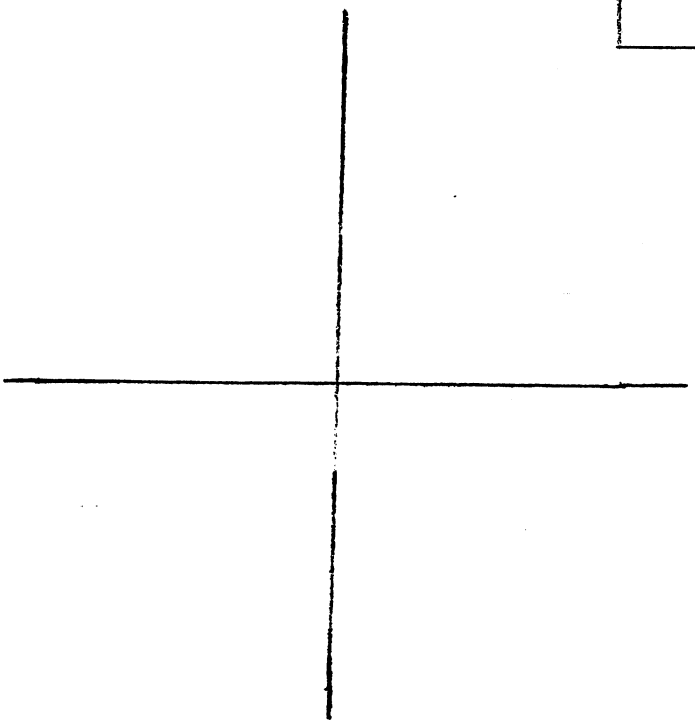
7. (20 pts) Find the area of the region that lies inside $r = f(\theta) = 1 + \cos\theta$ (a cardioid) and outside $r = g(\theta) = 2\cos\theta$ (a circle). You should express your answer as an integral, which you should not compute. Make a sketch of both curves on the graph provided below. Show any formulas you are using.



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