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Teaching Programs That Work

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During the past several years, a number of innovative programs have been established for helping students succeed in mathematics, particularly minority students. I have picked out a few to talk about, but you should be aware that there are many others and their number continues to increase. The ones I have picked represent a fair mix of type of student and of type of school.

SAN ANTONIO, PREP (Pre-Freshman Engineering Program) In 1979, Professor Manuel Berriozábal of the University of Texas at San Antonio founded PREP in an effort to encourage hispanic students who are talented in mathematics and science to continue through college with majors in those fields, rather than switch in high school or college to easier courses or drop out altogether. (I suspect he calls it an engineering rather than a mathematics program because it is easier to get money that way.) What he does is to get students from grades 6–11 and teach them a lot of mathematics and science to keep them interested. PREP has been expanding across the state, but I'll talk about the main program in San Antonio.

It is an academically intensive eight-week summer program; and students may return for a second or third summer. Admission and retention standards are strict. There is an orientation session for parents, most of whom come from backgrounds without traditions in higher education. There are daily classes in logic, engineering and computer science, algebraic structures, physics, probability and statistics, technical writing, and problem solving, as well as study sessions and guest speakers; in addition, there are occasional field trips and some practice with SAT tests. The guest speakers, from universities, industry, and government, give pep talks and describe career opportunities for math majors; this is of great importance, as students are usually unaware of such opportunities, part of the reason they switch to easier programs or drop out of school. Class size is in the low twenties except for emergencies. Faculty teach three courses each day and are on campus from 9:00 to 5:00. Over 600 students completed the 1989 program; three-quarters were hispanic and one in fifteen were black; half were from low-income families and received daily lunches, bus transportation, or stipends. I visited six classes and attended a guest lecture and gave one.

Many academic people derided PREP when it was first announced, maintaining that a mathematics professor would degrade his profession by being associated with it, that you can't get young kids to study logic for eight weeks, that minority students never do well in a traditional academic setting. In fact, the 1989 class voted logic the overwhelming favorite, with an interest rating of 98%. There are now 1100 PREP graduates of college age; of the 750 who responded to the latest survey, all have finished high school (whereas only 50% of students nationwide go past the 10th grade), and 90% are actually attending college or have graduated. Only 50% of freshmen in the Texas public colleges go on to finish, with less than 20% in science or engineering; among PREP graduates, 75% of college freshmen go on to finish, with 67% in science or engineering.

ESCALANTE Everyone in this room has heard of Jaime Escalante and his incredible successes with AP calculus at Garfield High School in East Los Angeles. Probably most of you have seen the movie *Stand and Deliver*. You should also know that there is a book *Escalante* [1]. It too is somewhat romanticized, but it includes gripping details not given in the movie.

Garfield High School has 3500 students, of whom 95% are hispanic, predominantly from middle or lower income families in which neither parent has completed high school. Eighty percent of the children qualify for the federal free or reduced-cost lunch program. Escalante arrived there in 1974. He had taught mathematics and physics in his native Bolivia, and he had ideals and standards. What he found was debris, graffiti, and gang fights; students who were surly, bored, unruly, and hostile, wedded to a life of academic failure; and teachers who didn't seem to care whether or what the kids learned. The basic mathematics text was 5th-grade level by Bolivian standards. To win students over, Escalante devised gadgets, gimmicks, and a special vocabulary, all of which he still uses as he entertains, challenges, cajoles, encourages, praises, warns, scolds, and threatens in English and Spanish and two Bolivian dialects. He plays music, sometimes soft, sometimes loud. He hands out candy, has the class chant and clap, and tells jokes. "Red light" means stop and think; "green light" means smooth sailing ahead. He squeals in ridicule of the marching band—a waste of time away from mathematics—and barks on behalf of his toy bulldog, which he asserts is 45-carat gold. (The bulldog is the school mascot.) A poster warns, "You don't do your homework, you gonna be working the rest of your life at Jack-in-the-Box."

Escalante also wins students by example. They respect a teacher who is at the school from 7:00 a.m. to 7:00 p.m. Even students not much interested in math work hard for him when they see him working so hard for them. He also challenges student athletes to handball: "You choose. I use left hand or right hand. You beat me, you get an A. I beat you, you do this homework" [1]. The student does the homework. Finally, Escalante provides personal support by accepting telephone calls at home, interceding with parents, arranging field trips, and organizing expeditions to McDonalds; even a clean-up-and-paint squad for the classroom or a money-raising car wash enhance a student's feeling of participation and self-esteem.

Now that Escalante is established, his showmanship is needed less if at all. Still, many students go for it. As one of them told me, "He's funny. He's very funny. He's creative when he teaches. He'll make an opening monologue, very fast, and he'll start telling jokes, and then all of a sudden he's hit you with a quiz—and you want to do it because you're in a good mood. He comes up with some different stuff every time you go to class: we go to class and there's some apparatus there. He gets you interested." I visited two of his classes, an algebra and a calculus. (He told the algebra class the bulldog cost him $1000; for the calculus students, he revalued it at $2000.) I also visited two classes of Ben Jiménez, his colleague: a trig and a calculus. The four classes averaged about 23 students. Jiménez is the exact opposite in character: quiet, undramatic, and no-nonsense. He doesn't use gadgets, nor even "red light" or "green light," but he achieves the same results. Like Escalante, he emphasizes on day one that he is a serious teacher teaching a serious course; that the students are going to have to attend class, study hard, and do a great deal of homework; and that the standards will be uncompromising. At the same time he makes clear that he is the students' friend; that he believes in them; that he is available in his classroom or adjoining office from 8:30 on, in particular during the noon hour and after school; and that he will work hard to help any student who is working hard.

Garfield students tackled AP calculus for the first time in 1979 and gained national fame in 1982. The AP was not an end in itself but a lofty, almost impossible challenge. The mere act of studying for it, whatever the result, was itself a worthwhile discipline. Those who passed acquired a strong sense of achievement for having performed something difficult as measured by a national standard. The results at Garfield started out modestly but picked up to an astounding record. In one 6-year stretch involving 250 students, the passing rate was 89%, vs. a national average of 70%. In the past few years, one quarter of all hispanic students in the country who passed calculus AP have come from Garfield High. Did you hear that right? *One quarter in the country came from Garfield.* (One may wonder what this says about the rest of the country.)

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What do AP students do for an encore? Fifteen of the eighteen students in the famous class of 1982 entered college and by 1987 nine had graduated—well above average for hispanics; several held professional or technical jobs. One had passed her CPA exams and another had completed her MBA. Equally notable is the way the spirit of success carried into the rest of the school: during those five years, the percentage of Garfield High School students admitted to college increased from 60% to 70%.

UC BERKELEY, PDP (Professional Development Program) The freshman class at Berkeley is 20% hispanic and 10% black. PDP is an honors program for freshmen, inaugurated in 1978 by Uri Treisman, who was concerned about the high rate at which black students with strong high school records in math were failing freshman calculus; moreover, only one in eight of those who did pass got through sophomore differential equations or pre-med organic chemistry, the prerequisites for careers in engineering and the sciences. (In a tragic twist of irony, when many of the students switched to other majors and graduated, the university counted them as successes, whereas they thought of themselves as failures.) To discover the roots of the problems, Uri made a detailed study of the backgrounds and study habits of the black students. Those from predominantly black high schools typically had less exposure to mathematics than other Berkeley students, causing them to overestimate their understanding of course concepts; those from predominantly white high schools also had trouble adjusting as they discovered to their dismay that they were not welcomed by Berkeley's white students. Uri observed that the black students tended to kept their academic and social lives separate. They almost invariably worked alone, with all the attendant frustrations that come from having no frame of reference; in one group he observed, only two of the students provided a counterexample by regularly studying with others: with each other—and at the end of the year they married and quit college. Chinese students, in contrast, studied in groups, exchanging hints and tips and offering constructive criticism of one another's work.

PDP is an intensive, demanding program for talented students, particularly minority students, who are planning a career in a mathematics-based profession. It is designed to help them excel at the university. Candidates are selected by an elaborate process that ensures an ethnic mixture, diversity of high school backgrounds, and gender balance. They are told that they are among the most promising freshmen and that the program is seeking students with a deep commitment to excellence and the desire to continue to graduate school and become leaders in their profession and in society. Several of Escalante's graduates have entered Berkeley and participated in the program. The program itself consists of enriched, intensive work sessions to replace the regular calculus recitations. The emphasis is on students' strengths rather than weaknesses—a student with an identified deficiency will be handed difficult problems where it will have to be met head on—the direct opposite of tutoring or other remedial programs. There are two 2-hour intensive sections per week, 15–20 students to a section. Students come in having already done the regular class homework and are handed worksheets containing challenging problems of the sort that separate A-students from B-students. They begin working the problems individually, then, when things get tough, in collaboration with one another. These experiences lead to a strong sense of community and the forging of lasting friendships.

Section instructor Duane Cooper (seated, second from left) encourages students at UC Berkeley's PDP to rethink the challenging problem before them. Professor Henry Gore (standing, fourth from left) of the Department of Mathematics at Morehouse College observes their interaction. Photograph courtesy of Jane Scherr.

The session is conducted by a "facilitator," a TA who guides the work but does not give out answers. An important goal is to ensure that the students will go on to excel in their sophomore and later courses without the program there to help them—to become "independent but not isolated learners," as the PDP people put it. The facilitator is a role model and peer counselor, and together with other program staff is sensitive to warning signs such as a distracted appearance or nonattendance; they act as a support group, intervening to help solve outside problems (housing, for instance) before they have become crises. (In contrast, university counseling offices usually see students only after they are already in trouble.) Activities such as pizza-parties and volleyball, as well as the informal atmosphere of the sections, provide additional social contacts that help counter possible feelings of isolation. This combining of the academic and social functions is regarded as an important feature of the program.

The proportion of black students who graduated or are still in school after six years or more is 39% for non-PDP students and 65% for PDP. There are now more than 30 satellite programs across the country. I visited the new ones at Cal Poly (San Luis Obispo) and UT Austin, as well as the main one at Berkeley, speaking with students, facilitators, administrators, and faculty, and sitting in on work sessions. The excitement of the students as they shout and argue about mathematics is a joy to behold. At Austin the Calculus 1 students in the program averaged 21 points higher than the rest of the class. Two years ago, a black female student from PDP became the first Rhodes Scholar at Berkeley in twenty-four years.

NEW MEXICO CALCULUS PROJECT New Mexico State University at Las Cruces is conducting an intriguing calculus project. About 30% of the students are hispanic, and there are satellite programs at several nearby schools, of which one is predominantly hispanic. The goal is to improve calculus teaching for all students by means of student research projects. It stems from a 1987 experiment by two young faculty members, Marcus S. Cohen and David J. Pengelley, whose aim was to get students "to discover the excitement of calculus, build their self-confidence in theoretical thinking, and thereby fundamentally alter their perception of what doing mathematics is really all about." For the formal project, they were joined by colleagues Edward D. Gaughan, Arthur Knoebel, and Douglas S. Kurtz. A research project is a calculus problem, more elaborate and challenging than a standard problem. Ten calculus sections (of the twenty in the college) use these projects, which replace the three one-hour exams. The instructors are the above five and five TAs; each TA is assigned to one of the professors and teaches a section of the same course (1st, 2nd, or 3rd-term). Help labs staffed by the TAs are open two to four hours daily during the two weeks of a project. Students are expected to turn in polished solutions, and, later, explain them in a private 15-minute interview with the professor. With two sections of 40 students, an instructor spends a lot of time reading papers, holding office hours, and interviewing. (Creating the problems in the first place is also time-consuming, but once a data bank has been established, that will no longer be a big consideration.)

A problem often involves simply fitting several concepts together, as when computing the volume cut from the first octant by a tangent plane to a given surface. Sometimes students have to consult a section of the text that the course did not cover. They may be asked to develop a topic on their own, such integration by hyperbolic substitution. Some problems are stated without numbers or variables, such as the following greenhouse problem, which causes their jaws to drop:

"Your parents are going to knock out the bottom of the entire length of the south wall of their house and turn it into a greenhouse by replacing some bottom portion of the wall by a huge sloped piece of glass (which is expensive). They have already decided they are going to spend a certain fixed amount. The triangular ends of the greenhouse will be made of various materials they already have lying around. The floor space in the greenhouse is only considered usable if they can both stand up in it, so part of it will be unusable, but they don't know how much. Of course this depends on how they configure the greenhouse. They want to choose the dimensions of the greenhouse to get the most usable floor space in it, but they are at a real loss to know what the dimensions should be and how much usable space they will get. Fortunately they know you are taking calculus. Amaze them!"

Only the better students can manage this type; so most problems proceed by a series of hints. Whatever the form, the work is intended to require reasoning over a period of days and so reward good thinkers rather than good test takers. A problem that wows the students is to show that the series $1 - 1/2 + 1/3 - 1/4 + \cdots$ can be rearranged to converge to any number: their invariable reaction is that it is not possible, and (to quote the faculty) deriving the result "revolutionizes their view of mathematics."

The program has been underway for only a year and a half, but the experimenters are highly encouraged by the anecdotal evidence and preliminary data and in fact amazed by what some students can do. During a long conversation I had with them about the program, they had nothing but praise for their students—in sharp contrast to the universal pastime of ridiculing or grousing. Many students find the projects exciting, but some of the weaker ones do not enjoy the challenges, and require lots of help. In my view, if a lot of students are learning to think more mathematically and enjoy it at the same time, the program is worth while; the weaker ones can always opt for the traditional course.

SUNY POTSDAM MATHEMATICS PROGRAM The mathematics program at SUNY College at Potsdam was started in 1969 by Clarence F. Stephens, who had just joined the department as chairman. Potsdam is a liberal arts college of 4000 students, located in the upper reaches of NY State. 95% are from New York high schools, primarily from lower middle class backgrounds, often from farming communities and small villages. The mean combined SAT for mathematics majors is about 1100; for the entire college, a bit less. Practically all the students are white, but Steve (as he is known to his friends) had tested and refined his ideas during twenty-two years at Prairie View A&M in Texas and Morgan State University in Baltimore, where the students were predominantly black. The program at Potsdam soon started bearing fruit but did not become widely known until 1987, when John Poland's article appeared in the MONTHLY [2]. Steve retired the same year, and Vasily C. Cateforis, his colleague and former student, became chairman. Perhaps the best way to describe the program is to read from a letter Steve wrote me last spring:

"You may be disappointed with the description of our program. We have not used grant money to delevop it, and the only innovation in the curriculum is our BA-MA program established in 1970, which provides an opportunity for able mathematics majors to earn both degrees in four years without attending summer school. Most efforts to improve undergraduate mathematics programs focus on curriculum and educational technology. While we acknowledge the importance of these two factors in the improvement of mathematics education, we focus on the human factor of changing students' perception that mathematics is an almost impossible subject for students to learn and only the most gifted can be expected to achieve any degree of success. We simply established a humanistic academic environment for learning mathematics in which students in mathematics courses feel good about themselves and find enjoyment in the study of mathematics as a result of proper teaching strategies and a supportive environment which promote student success and academic excellence. We help our students understand the meaning of a mathematical proof and have respect for it, to learn how to learn mathematics, to read a mathematics textbook with understanding and pure enjoyment, to study independently and as a member of a group. We use many different methods of teaching undergraduate mathematics. We teach in the spirit that *Everybody Counts*."

Clarence F. Stephens, founder of the mathematics program at SUNY College at Potsdam (1969). His determination to develop a flexible mathematics program that emphasizes the "human factor" has fostered marked success—a substantial number of Potsdam's students major in mathematics and those that concentrate in the discipline compose over 40% of the College's honors students.

The major at Potsdam consists of 30 to 40 hours of mathematics (out of 120): calculus (12 hours), set theory and logic, linear algebra, modern algebra, advanced calculus, a problem seminar, and electives. Class size is 40 at the lower level, 30 to 35 at the upper, 15 in the problem seminar. The predominant spirit is the culture of success: continual encouragement, recognition of every accomplishment, successful role models—honors students, BA-MA students, graduates—enough success to develop self-esteem, enough time to develop intellectually, recognition of one's achievement, and the belief that the study is worth while. Instead of racing through a long syllabus that students are largely not going to absorb anyway, the faculty want their students to learn enough of the subject well enough to understand the essential idea and general strategy. The students solve very hard problems in small groups, teaching one another as the professor guides the effort with helpful questions. Tests are regarded as articles of learning rather than measures of ability. Grading is flexible, to allow for late bloomers. Teachers focus on developing the students' skills rather than on the transmission of knowledge. They challenge the students—but within reason, consistent with Steve's maxim: "Teach the students you have, not the students you wish you had." No one motivates by threat. No one says "I taught them, but they didn't learn it"—that would be likened to a salesperson saying, "I sold it to them, but they didn't buy it."

The faculty believe that the best basis for understanding mathematics and its wide applications is experience in classical mathematics with its emphasis on logical structure, precision, careful analysis, and clarity of expression. Consequently, there are no service courses; nor are there remedial courses, nor even placement tests. (Have faith in them: throw them in and they'll probably swim.) There are no special mathematics courses for math edu-

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cation majors. There is no course titled "Calculus for two-headed football players." The only special course is honors calculus.

There is no mathematics requirement for graduation—but half the freshmen class take calculus as an elective. There is a large and flourishing Pi Mu Epsilon chapter. Although nationally, post-calculus courses account for only 10% of mathematics credits, at Potsdam the figure is 50%. While nationwide only 1% of graduating seniors are math majors, at Potsdam it is 22%. The proportion of women among all graduating seniors is 51% nationally and slightly higher, 54%, at Potsdam; but the proportion of women among graduating math majors is 46% nationally and 55% at Potsdam—although the department makes no particular effort to attract women into mathematics. As the mathematics program has prospered, it has attracted better and better students; by now, over 40% of honors graduates are math majors. Thirteen of the last 16 valedictorians were mathematics majors; 8 of the 13 were women. Did you get that? *Half the valedictorians were women mathematics majors.* I spent two-and-a-half days on campus, where I visited five classes and conferred at length with students and faculty and with the top administration—who, I am happy to report, support the mathematics program unequivocally. Again the faculty have nothing but praise for their students.

Most of the mathematics graduates go into industry, many to places like Kodak and IBM, or various insurance companies, where they prove to be able to think independently, read and write technical reports, work cooperatively with others, present and defend their work, and offer criticism constructively; many rise to high managerial positions. Others go on to do graduate work in mathematics-related fields, often at Cornell or Big Ten schools, where they find themselves well prepared for independent work despite a mathematical education that may be less broad than that of other students.

All the departmental faculty have engaged in research, all teach courses at every level, all are dedicated to the program. There are no TAs. The faculty work long hours, subordinating any research ambitions to the success of their students. The dean told me of the time he was walking across the campus late one Friday afternoon when spring had just broken out and everyone had fled home early—except that as he passed the mathematics building he happened to look up and there were the professors still at work at their desks.

Some mathematicians believe only potential researchers should major in mathematics. But this would write off a mathematically educated citizenry. I for one hope to see many more Potsdam graduates sitting in our state legislatures when university budgets next come up.

CONCLUSIONS All these programs are based on teaching strategy rather than on curriculum reform. The fundamental precepts are: challenge your students with difficult problems, demand hard work, and adhere to uncompromising standards; at the same time, constantly promote and bolster their confidence and self-esteem, assuring them that they can succeed and praising each accomplishment, and make yourself available outside of class for sympathetic help and encouragement. "Remediation" is a dirty word—in fact, a dirty concept. Classes are kept small. Where the program is part of the regular teaching assignment (Garfield, New Mexico, Potsdam), the faculty generally work long hours at their teaching.

The principle of having students working in groups is basic to the Berkeley and Potsdam programs. (I permit myself the irreverent observation that when students learn from one another they take up less of *your* time.) Berkeley maintains a support structure to help students adjust to the new environment and to help keep nonacademic problems out of the way—important for minority students just entering the university. Potsdam tenders support via its Pi Mu Epsilon chapter and other activities typical of a big happy family. Escalante provides his personal support.

I learned something about role models. I asked Escalante, "Suppose there was a teacher who was your clone except that he was anglo; would he have the same success with these students?" I expected him to say, "No, he has to be hispanic." Instead, he said, "No. I pronounce their names right," adding, in explanation, "It comes from within." He went on to say that an anglo who lived for several years in Mexico, say, and absorbed its culture, could also succeed. Then he added, "I could teach an anglo class." The story at Potsdam, with its incredible record of female math majors, is even more interesting. Professor Pat Rogers of York University recently concluded a detailed study of the department, in which she states: "Given the importance placed by some writers on providing female students in male-dominated fields with female role models, it surprised me to find that in a department of 15 faculty, only one is female [3]." The women students spoke of their male teachers as father-figures and, in fact, role models, and were unaware that females are often discouraged from taking math. The article presents an interesting discussion of power and other social or psychological factors and their relation to the nature of mathematics and concludes that apparently "in an environment which is genuinely open to and supportive of all students and in which the style of teaching is true to the nature of mathematical inquiry, women are attracted to mathematics and are just as successful as men."

Here then are five programs with success records ranging from substantial to unbelievable. I'll temper that. I have not defined success. I do not agree with every detail of every program. Some reports are skewed, as when they are limited to those who took the trouble to respond to a survey, or when they do not separate the effect of the program itself from the fact that its students were preselected high achievers. I picked up occasional infelicities on the part of lecturers, such as the statement $f(x) = x^2 - 3x = 2x - 3$, which the students dutifully copied. (What about $3x = 21 = 7$? Or $3x = 21 = 8$?) But my statement stands. The problems we face in educating our youth, especially minorities, are daunting, and it behooves us to study these programs and learn from them.

Before concluding, I wish to express my thanks to Reba Gillman, who is my wife, and Jackie McCaffrey, the coordinator of UT Austin's "Berkeley" program, for many insightful comments on earlier drafts of this talk.

There is one common feature of these programs I have not yet mentioned—perhaps the most outstanding one of all. Can you guess what it is? They were all created by dedicated, imaginative individuals, working by themselves. *MAA had nothing to do with it.* Instead, we were sitting around in committees talking about—oh, never mind!

Finally, it appears that good teaching is good teaching, whether your students are above or below average, live in poverty or in affluence, or represent any particular color or sex.

This article is Professor Gillman's Retiring Presidential Address, presented January 19, 1990 at the MAA meetings in Louisville, Kentucky.

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Photograph of UC Berkeley's Professional Development Program (PDP) on page eight courtesy of Jane Scherr, Berkeley, California.