Teaching Problem Solving and Writing Explanations

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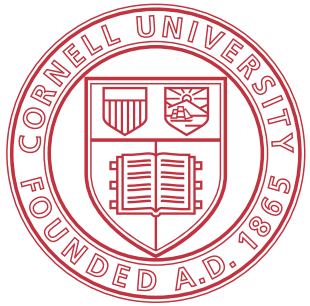

Welcome!

Aims of this workshop:

- To provide a general framework to use when helping our students learn how to solve problems.
- To share strategies we have used in the past to teach problem solving, and analyse how helpful these have been.
- To discuss helpful ways to make our students self-sufficient and give them a basis for analysis, not just in mathematics.

I don't claim to know the best ways to teach problem solving, I'm here to give some structure to the discussion. No one has all the answers, we are all here to share and discuss, and do the best we can by our students.

Motivating the need

We need to make it clear to the students why explanations and proofs matter.
What are some practical ways to do this early in a course?

- Give an example of how justification and problem-solving arise in real life.
- Demonstrating what a bad explanation looks like.
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These two topics are linked: writing good explanations makes it much easier to solve problems!

Problem Solving: Identifying where students get stuck

Common statements from students when asking for help with a given problem:

1. "I don't know how to get started."
2. "I've tried these things but none of them worked."
3. "I've got the idea, but I don't know how to put it down on paper."

These statements match up quite nicely with Pólya's problem solving strategy.

Pólya's Problem Solving Methods

George Pólya

In his book "*How to Solve It*", Pólya lays out a general strategy for how to approach (maths) problems, following four phases:

1. Understanding the problem
2. Devising a plan
3. Carrying out the plan
4. Looking back

His method of helping students relies largely on *asking the right questions*.

Good problems

"If the student is lacking in understanding or in interest, it is not always his fault; the problem should be well chosen, not too difficult and not too easy, natural and interesting, and some time should be allowed for natural and interesting presentation."

We need to make sure that our problems are chosen carefully: that they are interesting and approachable for the student, and they are pedagogically worthwhile.
What aspects should a good problem embody?

Understanding the problem

"I don't know how to get started"

Good prompts reach a balance between being general enough that the student can re-use the questions on other problems and become self-sufficient, yet not so general as to be unhelpful (e.g. "just think about it harder").

An example, often used by Pólya, is a kind, but firm "What is the unknown?"

Understanding the problem

Consider the following problem:

I decided to bake some brownies in a rectangular pan for my two young cousins. They are obsessed with justice and fairness, and as such I need to make sure that they get exactly the same amount of brownies. To make things more difficult, my housemate comes by while they're cooling on the counter and cuts a rectangle out of the middle - the rectangle could be anywhere, and its sides are not necessarily parallel to the edges of the pan. How can I cut the rest of the brownies to be sure that both my cousins get exactly the same amount?

What are some helpful questions or prompts for a student struggling to start the problem?

What are some unhelpful questions for a student struggling to start the problem?

Understanding the problem

General questions to help the student understand the problem:

Devising a plan

"I've tried these things, but none of them work"

A first step in devising a plan is being confident that we are capable of doing so. We're less likely to make a substantial effort if we don't think we will ever succeed.

"Determination fluctuates with hope and hopelessness, with satisfaction and disappointment. It is easy to keep on going when we think that the solution is just around the cornerl but it is hard to perservere when we do not see any way out of the difficulty."

How can we introduce material and problems in a way that students recognise that they are capable of working on a problem?

How can we approach learning in the classroom to encourage students to try to solve a problem, or that it's not bad to have a strategy fail?

Devising a plan

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Devising a plan

How can we help our students devise a plan for the brownie problem?

General questions and suggestions to devise a plan to solve a given problem:

Carrying out the plan

"I've got the idea, but I don't know how to put it down on paper"

"To carry out the plan is much easier; what we need is mainly patience. [...] The main danger is that the student forgets his plan."

At several points during the process, it's helpful to ask: what is the question asking for? What are we trying to do? What are we aiming for?

This is also the step where writing explanations is key: checking each step is difficult if each step isn't clearly written.

Carrying out the plan

How to deal with getting stuck

There are two major reasons for being stuck:

- Not knowing what to do next
- Being on the wrong path

It is difficult to determine which of the two options is causing us to be stuck, but we can try to prompt our students to ask themselves questions to either give up on the current strategy or try something new within the strategy.

What are some questions and prompts to help students decide?

Looking back

The forgotten step

This is arguably the most important step in developing our students' problem-solving skills: reviewing what they have learnt, seeing what could be removed or simplified, and allowing for a development of mathematical curiosity: what could we change in the problem? How will this affect the outcome?

Have you emphasised this aspect of problem solving in class or in office hours? What questions did you ask? How did this lead to further or deeper discussion of the material?

Could we add this to our assessment? (cf "Teaching Problem-solving in Undergraduate Mathematics")

Conclusion

- Prompting questions should be general and reusable.
- Explaining their work is an important step for students on the road to self-sufficiency in problem solving.
- We need to build up our students' mathematical self-confidence.
- Looking back on a problem is an often overlooked step that is vital in consolidating understanding.
- What teaching workshops would you like?